ARTICLE IN PRESS

Journal of Business Research xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

Journal of Business Research

journal homepage: www.elsevier.com/locate/jbusres

# A complex networks approach to pension funds

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ARTICLE INFO

*Keywords:*
Pension funds
Benchmarks
Complex networks
Community detection

ABSTRACT

In this paper, techniques proper to complex networks studies have been applied to analyze features of the investment styles and similarities in the Italian pension funds. The analysis has been developed through interdisciplinary approaches. First, we look at the node degree distributions; next, we consider the centrality measures, like betweenness and closeness. Results indicate that the network of funds is dense and assortative, with short path lengths. Moreover, through community detection algorithms, it is found that many funds show similar features. In particular, the network of benchmarks is far from being dense, is characterized by hubs, and is disassortative. Furthermore, the insertion of weights does not produce dramatic changes in the centrality measures, but it blurs the communities. Still, the *k*-core and the highest *k*-shell do properly evidence the most popular benchmarks. In conclusion, the network structure of the Italian pension funds, without taking into account information from weights, seems to contain already sufficient information for detecting similarities in investments styles.

## 1. Introduction

The pension funds importance at country level can be measured by comparing the amount of pension assets to the GDP level. In the last annual report on pension funds, OECD (2018a) highlights that pension assets, in the OECD area, achieved a record USD 43.4 trillion in 2017. U.S. and U.K. hold the largest amount of pension assets, 64.9% and 6.7% of the total assets in the OECD area, respectively. In Italy, pension assets accounted for 9.8% of GDP in 2017, well below the average in the OECD area (50.7% of GDP). However, they still are a quite remarkable segment of the domestic market. OECD (2018b) has emphasized the need of providing adequate risk management and risk monitoring system in the Italian pension funds with respect to their investment policy, - following the issue in 2014 of the new regulation on investments which has relaxed some quantitative limitations.

Despite its economic importance, the literature on the performance and the structure of the pension funds market is still very limited. One of the main reasons lays in the paucity of available data on the pension funds features and investments returns. The main research contributions focus on the pension fund management and the connections between managers. Blake, Rossi, Timmermann, Tonks, and Wermers (2013) analyze the decentralization in investment management in the U.K. pension funds industry in the period 1984–2004, using a data set including information on quarterly returns and on the type of mandate. Blake et al. (2013) provide models for both centralized and decentralized management in order to provide predictions on the economics of pension fund decentralization.

Rossi, Blake, Timmermann, Tonks, and Wermers (2018) analyze the connection between fund sponsors, fund managers, and investment consultants of defined benefit pension funds in U.K. Their findings give evidence of the relation between fund managers network position and investment performance.

Ding, Parwada, and Shen (2017) build the networks of delegated portfolio managers, basing their study on investment mandates between pension plan sponsors and their sub-advisors. The same paper examines the role of the plan sponsor in the investment mandates, which, in turn, produces the effect of information transmission among investment firms. Sharing the information within mandate networks results in a lower diversification benefit.

Our paper aims at analyzing interrelations among Italian pension funds from their exposure to benchmarks. The methodology is based on complex networks considerations and consists in the construction of a bipartite network in view of detecting overlaps of pension fund styles.

The pension funds investment behavior may be affected by both their strategic decisions and other factors depending on the reference institutional framework. In Italy, the reference framework is the strategic asset allocation based on the benchmark. In the field of pension funds, the strategic asset allocation is the result of the combined actions

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https://doi.org/10.1016/j.jbusres.2019.10.071
Received 14 March 2019; Received in revised form 29 October 2019; Accepted 30 October 2019
0148-2963/ 

Please cite this article as: Anna Maria D'Arcangelis, Susanna Levantesi and Giulia Rotundo, Journal of Business Research, https://doi.org/10.1016/j.jbusres.2019.10.071

of pension fund managers and pension fund members and is consistent with the nature and duration of the expected pension liabilities. The percentage of risky or safe assets, or more generally the choice of the investment mix, should depend on individual parameters (such as the age composition of the fund's members) as well as on the relationship between investment risks and the maturing of the pension fund.

Our approach is quite different from the current literature. In fact, we focus the analysis on the similarity among the investments according to the selection of the benchmarks.

The usage of complex networks for understanding relationships between mutual funds is still at its beginning. D'Arcangelis and Rotundo (2015) relate the network structure to the performance of (Italian) mutual funds. Li, Ren, Feng, and Zhang (2016) build a one-dimensional network based on common shareholders for the China stock market, and suggest economic mechanisms for the relevance of the network structure within this stock market. Further studies, taking other sources of information beyond financial time series, have been also examined from networks perspective. For instance, the interlock of directorates evidences the relevance of information not necessary encompassed in other datasets (e.g. Grassi, Patarnello, & Szpilska, 2008; Rotundo & D'Arcangelis, 2010); geographical location has been also considered by Rotundo and D'Arcangelis (2016) for a classification of the overlaps. The detection of similar asset holdings has been proved to serve as explanatory variable for future returns and fund flows by Blocher (2013).

In general, the presence of short connections and high overlap creates the condition for an eventually quick propagation of contagions cascades (Elliott, Golub, & Jackson, 2014). To the best of our knowledge, aside the contribution of Braverman and Minca (2018) on mutual funds, no research paper has attempted to provide insights on the overlap of pension funds holdings using complex networks methodologies.

Despite some relevant common aspects (e.g. the category of investments and the portfolio management services offered), mutual funds and pension funds have some important differences: mainly the different time horizon of the investments (longer for pension funds) and the financial need they have to fullfill. These aspects affect the choice of the standards to evaluate their investment strategies. In brief, mutual funds with high performance are able to attract high cash inflows. Fund managers compensation is linked to both the size of the fund (assets under management) and the excess of fund performance over the benchmark. Therefore, this can constitute a strong incentive for mutual funds managers to carry out an active management policy. Otherwise, pension fund inflows are influenced by demographic and actuarial elements, instead of assets performance. The longer time horizon of pension funds investments leads managers to invest also in illiquid assets. Moreover, the dimension of the pension funds assets under management is related to the number of plan members and to a specific regulation that goes beyond efficiency considerations (Andonov, Bauer, & Cremers, 2011).

In Italy, pension funds are organized in four categories: "contractual pension funds", "open pension funds", "pre-existing pension funds" and "individual pension plans". The first one collects the contractual pension funds, that are legally autonomous bodies intended to specific categories of workers, e.g. private sector employees belonging to the same contractual category, same company or group of companies; public sector employees; self-employed workers; etc. These pension funds can be set up on the basis of collective agreements, including corporate agreements, signed by the employers' association and workers. In the absence of collective agreements, through company regulations, agreements between cooperative workers members or between group of participants promoted by the trade unions. Membership is voluntary and open only to employees that meet the conditions established by the agreement.

The second category concerns the open pension funds which are set up and managed by banks, insurance companies or investment management companies for generic groups of workers (self-employed workers, employees, etc.). Membership can be individual or collective, if the fund regulation allows it. These funds are not independent legal entities and pension assets are legally separated from the entity which manages the fund and, as such, cannot be subject to enforced execution by the sponsor company's creditors.

The third category refers to the pre-existing pension funds that are funds already operating before the entry into force of Legislative Decree 124/1993. After 1993, these funds have maintained their structure without undergoing real changes, as the new legislation does not concern them. They can be legally autonomous or non-autonomous bodies directly managing their assets. These plans are both defined contribution and defined benefit plans. The latter are closed to new members. Finally, the last category refers to the individual pension plans realized through life insurance contracts and named PIPs.

Therefore, we investigate the overlap of Italian contractual and open pension funds measured by their exposure to benchmarks. The analysis focuses on equity sub-funds. Data were provided by MEFOP[1] and cross-checked with the Bloomberg database.

The paper is organized as follows. Section 2 provides a description of pension funds investments. Section 3 describes the dataset. Section 4 outlines the method of analysis through complex networks approaches. In Section 5 we discuss the results. Conclusions follow.

## 2. Pension funds investments

This section provides a description of the pension funds investments and overall information about portfolio allocation and members by category of investment in the Italian private pension markets. Each category of pension funds is structured in accord to their investments profile. The main difference is between single-strategy sub-funds and multi-strategy sub-funds. The former offers a single investment profile to their members, the latter proposes several investment options characterized by different risk/return profiles to their members.

The Italian pension funds supervisory authority (COVIP) classifies the investment sub-funds according to the type of assets the fund invests in. This classification is listed below (Covip, 2018):

- *Bond sub-funds* (and other debt instruments): consisting of bonds; investment in equities is not allowed. They are suitable for pension fund members with low risk appetite, aiming to achieve capital gains in the medium term.
- *Mixed bond sub-funds*: sub-funds in which the investment in equities is less than 30% of total asset. They are suitable for pension fund members with medium risk appetite.
- *Equity sub-funds*: sub-funds in which the investment in equities is greater than 50% of total asset. They are suitable for pension fund members with high risk appetite, aiming to achieve capital gains in the medium-long term.
- *Guaranteed sub-funds*: sub-funds in which the capital or a minimum return is guaranteed, regardless of the portfolio composition.
- *Balanced sub-funds*: based on a balance of risks among a range of asset classes. They are suitable for pension fund members with medium risk appetite, aiming at achieving higher returns than bond sub-funds, but being exposed to less volatility than equity sub-funds.

Table 1 shows the portfolio allocation of the Italian pension funds in 2017. They invest 58.1% of their assets in debt securities (sovereign bonds and other debt securities), 17.7% in equities, 14.4% in UCITS,[2] 1.6% in real estate and 0.9% in other assets and liabilities. The

[1] MEFOP is a public entity jointly owned by the Italian Ministry of Economy and Finance (which is the main shareholder) and most of the Italian pension funds. It was founded in 1999 by the Ministry of Economy and Finance and carries out institutional activities in the field of supplementary social security.

[2] UCITS (Undertakings Collective Investments in Transferable Securities) are harmonized European mutual funds, regulated by the European Union (Directive 2014/91/EU).

**Table 1**
Pension funds' asset allocation. For each category, the first column reports the economic dimension expressed in million of euros. The second column reports the percentages pertaining to each asset class. Year 2017.

| Asset class | Pension Fund category | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Contractual | | Open | | Pre-existing | | PIPs | | Total | |
| Cash and deposit | 3459 | 7.0% | 2333 | 12.2% | 2303 | 7.3% | 1182 | 4.2% | 9278 | 7.2% |
| Sovereign bonds | 22,569 | 45.6% | 7088 | 37.0% | 9490 | 29.9% | 14,059 | 50.5% | 53,272 | 41.5% |
| Other debt securities | 8669 | 17.5% | 1211 | 6.3% | 4595 | 14.5% | 6767 | 24.3% | 21,251 | 16.6% |
| Equities | 10,579 | 21.4% | 3780 | 19.7% | 5569 | 17.6% | 2744 | 9.9% | 22,672 | 17.7% |
| UCITS | 4222 | 8.5% | 4727 | 24.7% | 6801 | 21.5% | 2763 | 9.9% | 18,519 | 14.4% |
| Real estate | – | – | – | – | 2048 | 6.5% | – | – | 2048 | 1.6% |
| Other assets and liabil. | −42 | −0.1% | 30 | – | 886 | 2.8% | 330 | 1.2% | 1179 | 0.9% |
| Total | 49,456 | 100.0% | 19,145 | 100.0% | 31,692 | 100.0% | 27,845 | 100.0% | 128,218 | 100.0% |

**Table 2**
Contractual and open pension funds. Members and assets by type of investment sub-funds. Year 2017.

| Type of sub-funds | Number of funds | | Members (%) | | Assets (%) | |
|---|---|---|---|---|---|---|
| | Contractual | Open | Contractual | Open | Contractual | Open |
| Guaranteed | 37 | 48 | 23.8% | 20.7% | 14.7% | 14.7% |
| Bond and mixed bond | 21 | 53 | 18.0% | 11.5% | 26.8% | 33.4% |
| Balanced | 37 | 54 | 56.6% | 52.3% | 55.0% | 36.2% |
| Equity | 12 | 41 | 1.6% | 15.5% | 3.5% | 15.7% |
| Total | 107 | 196 | 100.0% | 100.0% | 100.0% | 100.0% |

remaining 7.2% of the portfolio assets are cash and deposit. Sovereign bonds and other debt securities report the exposure of pension funds to sovereign risk. A great attention is paid by the European Central Bank to sovereign debt markets, especially in the distressed markets. Sovereign risk has become a relevant topic among the EU countries, as sovereign bonds might lose the advantage of being risk-free assets due to the sovereign debt crisis. In the Italian market, the public debt securities are predominant and pension funds primarily investing in this category of assets are exposed to lower performance in case of a fall in domestic sovereign bonds prices. The UCITS mutual funds have become more popular among fund managers after the crisis of 2008–2009 (investors required higher level of protection after crisis) for the higher safety and flexibility offered. In general, due to their features UCITS are considered to adopt less risky strategies (see e.g. Camilleri & Farrugia, 2018). Table 2 shows the portfolio allocation of the Italian contractual and open pension funds in 2017. Both these categories of pension funds will be considered in the case study. Fig. 1 shows the distribution of the pension fund members (data from Table 2, columns 4–5) and portfolio assets (data from Table 2, columns 6–7) according to the different investment sub-funds in 2017. The pension fund members definitely prefer the balances sub-funds both in contractual and open categories. In view of so doing, they choose equity sub-funds if they are contractual funds and bond/mixed bond if belonging to the open funds category.

## 3. The dataset

In the present analysis, we consider only equity sub-funds, which show more frequent and substantial fluctuations. The exposure to equities is relevant for younger members of pension funds that are advised to subscribe the most risky investments profiles; in fact, they contain the highest amount of equities. Truly, they have a very long period of time which allows to compensate for the effects of the volatility of stocks. Instead, the older members, close to retirement age, subscribe less risky investments in order to preserve the accumulated capital.

Data have been provided by MEFOP and refer to year 2017. They are gathered in two datasets:

1. Equity sub-funds;
2. Self-declared benchmarks[3] with their associated portfolio weights.

The self-declared (or declared) benchmark is the parameter the fund managers announce to replicate (passive strategy) or to outperform (active strategy). Therefore, the pension fund managers should be able to achieve a strategic asset allocation performance in line with the results of the declared benchmarks.[4] The benefit of using self-declared benchmarks is to add insights on the geographical allocation and risk exposure of pension funds.

The investigated dataset includes 61 active sub-funds (belonging to 49 funds) with their 72 self-declared benchmarks. Data have been cross-checked with the Bloomberg database (Bloomberg, 2018). The pension funds in the dataset are identified by the following features:

- Fund name
- Fund category (contractual or open)
- Investment style (in accord to different risk profiles)

The pension funds sample is presented in Table 3 and Table 4 for contractual and open category, respectively. All the categories listed in the investment style refer to equity sub-funds and differ in the proportion of assets invested in equities (this proportion must however be greater than 50% of total assets).

Among the pension funds included in the case study, 31% are contractual and 69% are open. About 70% of the contractual pension funds is managed by asset management companies and the remaining percentage by stock brokerage company and private banking. The open pension funds are instead primarily managed by insurance companies (76%) or by asset management companies (24%).

Funds/sub-funds aim at achieving high performance by investing in diversified international assets traded on the main worldwide markets. Therefore, both contractual and open pension funds mainly invest in equities; as shown in Fig. 2 that illustrates their type of investments (bond, equity or monetary).

Table 5 shows the list of the declared benchmarks by geographic area; from this, we can deduce the pension funds diversification level through international investments. Note that the FTSE MIB is the leading benchmark index for the Italian equity markets. It includes highly liquid Italian firms belonging to the Industry Classification Benchmark (ICB) sectors and quantifies the performance of 40 Italian equities.

This list in Table 5 is a little bit different from the original one from MEFOP. We decided to bring together all the benchmarks, when the

[3] Benchmarks are commonly used financial indicators chosen in accordance with the investment policy adopted by the fund/sub-fund.

[4] In a performance attribution perspective, the fund global performance could be due also to timing and security selection activities (see Brinson, Hood, & Beebower, 1986; Brinson, Singer, & Beebower (1991)).

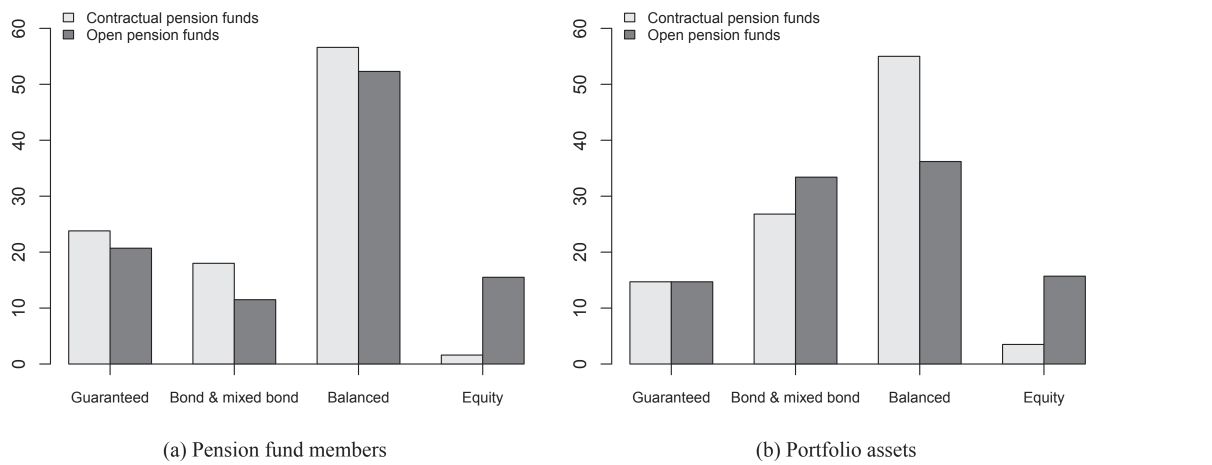


**Fig. 1.** Pension fund members (left plot) and portfolio assets (right plot) by investment sub-fund (percentage values). Year 2017. *Data source:* Covip (2018).

**Table 3**
The contractual pension funds sample.

| Fund name | Investment style |
|---|---|
| Alifond | Dinamico |
| Cooperlavoro | Dinamico |
| Eurofer | Dinamico |
| Fon.Te. | Dinamico |
| Foncer | Dinamico |
| Fonchim | Crescita |
| Fondaereo | Crescita |
| Fondaereo | Linea 4 (o D) – Prevalentemente azionaria |
| Fondaereo | Prevalentemente Azionario/Crescita |
| Fondapi | Crescita |
| Fondosanitá | Espansione |
| Fopen | Bilanciato Azionario |
| Fopen | Prevalentemente Azionario |
| Gommaplastica | Dinamico |
| Mediafond | Comparto Azionario |
| Mediafond | Dinamico |
| Previcooper | Dinamico |
| Previmoda | Rubino Azionario |
| Telemaco | Crescita (orange) |

Pearson correlation coefficient was greater than 0.90. The group with such high correlations has been merged into their reference benchmark. In the appendix (Table 8), we report the list of the original benchmarks involved (in the left column) and the reference benchmarks (in the right column).

The funds considered in this work spread their investments in geographical areas, mainly like in the Eurozone and North America. This behavior is in line with the OECD directives that encourage access to markets, limiting restrictions and widening the list of countries in which pension funds can invest (OECD, 2018a).

## 4. The network approach

The interest in complex networks has been constantly rising in the last years for studying mutual funds. A wide stream of literature focuses on the returns and their correlation (Garas, Schweitzer, & Havlin, 2012; Gligor & Ausloos, 2008; Varela Cabo & Rotundo, 2016). Indeed, data on fund holdings directly give insights on the overlap and dependence among portfolios. In this paper, we use complex networks to examine the structure of pension funds asset allocation. We expect that disentangling the contribution due to the similarity of investments from the bulk of information through the network structure contributes to evidence similar management styles and to identify variables that lead, in turn, to similar returns and high correlations (D'Arcangelis & Rotundo, 2015).

**Table 4**
The open pension funds sample.

| Fund name | Investment style |
|---|---|
| Allianz Previdenza | Linea Azionaria |
| AlMeglio | Azionario |
| Arti & Mestieri | Crescita 25 + |
| Aureo | Azionario |
| AXA MPS Previdenza in Azienda | Rettangolo |
| AXA MPS Previdenza in Azienda | Scaleno |
| AXA MPS Previdenza in Azienda | Sviluppo |
| AXA MPS Previdenza Per Te | Crescita |
| Azimut previdenza | Crescita |
| Azione di Previdenza | Dinamica |
| BAP Pensione 2007 | Investimento |
| Cattolica Gestione Previdenza | Azionario Globale |
| Core Pension | Core Pension Azionario 75% |
| Core Pension | Core Pension Azionario Plus 90% |
| Credemprevidenza | Azionario |
| Eurorisparmio | Azionario Europa |
| Eurorisparmio | Azionario Internazionale |
| Fideuram | Crescita |
| Fideuram | Valore |
| Fondo pensione aperto AXA | Dinamico |
| Fondo pensione aperto Bim vita | Equity |
| FPA Credit Agricole Vita | Linea Dinamica |
| Generali Global | Azionario |
| Giustiniano | Azionaria |
| Helvetia Domani | Azionario |
| Il Melograno | Dinamica |
| Insieme | Linea Azionaria |
| Insieme | Linea Bilanciata |
| Pensplan Plurifonds | ActivITAS |
| Pensplan Plurifonds | SummITAS |
| Pioneer Futuro | Azionario |
| Previd System | Rivalutazione azionaria |
| Previgest Fund Mediolanum | Azionario |
| Programma Open | Prevalentemente Azionario |
| Seconda Pensione | Espansione |
| Teseo | Sviluppo Etica |
| Unicredit FPA | Dinamica - Azionaria Internazionale |
| UnipolSAI Previdenza FPA | Azionario |
| Vittoria Formula Lavoro | Previdenza Capitalizzata |
| Zed Omnifund | Azionaria |
| Zed Omnifund | Bilanciata 65 |
| Zurich Contribution | Dinamica |

In general, a network is a collection of nodes and links, where the links represent a relation among couples of nodes. The network structure is described by its *adjacency matrix* $A = (a_{ij})$: $a_{ij} = 1$ if the element listed at row $i$ is connected to the element listed at column $j$; $a_{ij} = 0$ otherwise. The values $a_{ij}$ are named *weights* of the links most when they get values in a discrete or continuous set.

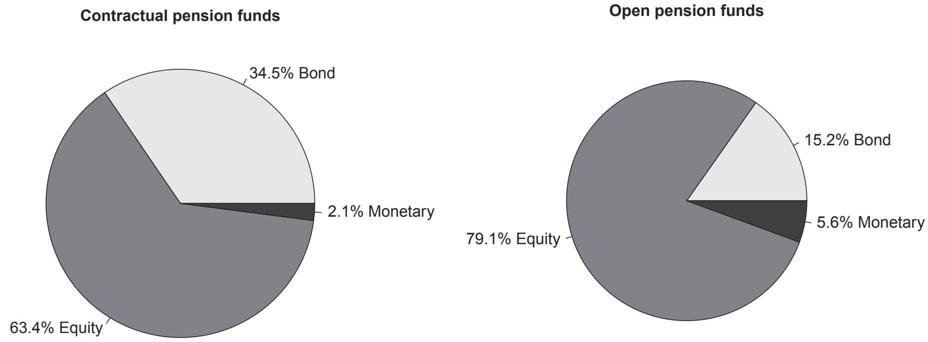


**Fig. 2.** Contractual and open pension funds by type of benchmark.

**Table 5**
List of the benchmarks by geographic area.

| Benchmark | Geographic area |
|---|---|
| 75% ISTAT + 2.5% | Italy |
| Barclays Capital Euro Inflation Linked Government Bond | Eurozone |
| Barclays Capital Euro Treasury | Eurozone |
| Barclays Capital Pan European Aggregate Credit | Europe |
| Barclays Capital US Credit Index | US |
| Barclays Euro Govt. Inflation linked All Markets ex Greece | Eurozone ex Greece |
| Bofa Merrill Lynch Emu | Eurozone |
| Bofa Merrill Lynch Emu Corporate | Eurozone |
| Bofa Merrill Lynch Emu Direct Government Bond | Eurozone |
| Bofa Merrill Lynch Euro Treasury Bill | Eurozone |
| Bofa Merrill Lynch Global Government Bond ex Japan | Global ex Japan |
| Bofa Merrill Lynch Italy Treasury Bill | Italy |
| Bofa Merrill Lynch Pan-Europe Governative | Europe |
| Bofa Merrill Lynch Us Treasury Note | US |
| Citigroup Euro Government Bond | Eurozone |
| Citigroup Eurobig | Eurozone |
| Citigroup World Government Bond Non Euro | World ex Eurozone |
| Comit Global | Italy |
| Ecpi Emu Governance Government bond | Eurozone |
| Ecpi Emu Governance Government bond Inflation linked bond | Eurozone |
| Ecpi Global developed Esg Best in class equity | Global |
| Ecpi Global developed Esg corporate ex financials bond | Global |
| Ecpi Global developed Esg corporate financials bond | Global |
| Ecpi Global developed ex Emu Governance government bond | Global ex Eurozone |
| Euro Stoxx | Eurozone |
| Euromts Eonia Investable | Eurozone |
| FTSE 100 | UK |
| FTSE MIB | Italy |
| FTSE Mts Eurozone Government Bond Investment Grade | Eurozone |
| JP Morgan Emu Cash | Eurozone |
| JP Morgan Emu Government Bond Investment Grade | Eurozone |
| JP Morgan Euro Government Bond | Eurozone |
| JP Morgan Europe | Europe |
| JP Morgan GBI broad traded | Global |
| JP Morgan Global Government Bond ex Emu | Global ex Eurozone |
| JP Morgan US Government Bond | US |
| Libid | Eurozone |
| MSCI Emerging Markets | Emerging Markets |
| MSCI Europe | Europe |
| MSCI North America | North America |
| MSCI Pacific | Pacific[1] |
| MSCI world | World |
| Mts Italy BOT ex-Bank of Italy | Italy |
| Mts Tasso Fisso Breve Termine | Italy |

[1] Five developed countries are included: Australia, Hong Kong, Japan, New Zealand and Singapore.

Here, both pension funds and benchmarks are represented by nodes. The information on pension funds has been gathered in the matrix $A = (a_{ij})$, where the $N$ rows report the pension sub-funds, and the $M$ columns refer to the benchmarks, listed in alphabetical order. An element of the matrix $a_{ij} = 1$, if the pension sub-fund at row $i$ invests in the benchmark at column $j$; it is 0 otherwise. Such an arrangement of the matrix $A$ represents a bipartite network, that is a network where there are no links among elements in the same group (the listed pension funds do no invest in other listed pension funds; a similar remark holds for the benchmarks). The 0/1 values of $A$ well evidence the choices for the investments, and it serves to answer the primary question on understanding the role of connections. Thereafter, such a matrix can be embedded in another standard adjacency matrix form $\widetilde{A}$ with size $(N + M) \times (N + M)$ reporting on both the rows and the columns the entire set of nodes:

$$\widetilde{A} = \begin{pmatrix} 0 & A \\ A^T & 0 \end{pmatrix}$$

where the 0s represent matrices with proper size. It is worth mentioning that *bipartite* networks are used in many other applications such as the coauthor networks (Lambiotte & Ausloos, 2005; Varela Cabo & Rotundo, 2016).

Key quantities in the analysis of complex networks are the so called *centrality measures* (CM), i.e. rankings among network nodes in accord to their relevance with respect to some specific criterion (Boccaletti, Latora, Moreno, Chavez, & Hwang, 2006). The oldest and most used CM is the node-degree, that is the number of links entering in (in-degree) or exiting from (out-degree) a node. Technically, the node-degree can be calculated as follows: given $e = (1, 1, \cdots, 1)^T \in R^M$, where $M$ is the number of benchmarks, $u = Ae$ calculates the out-degree (of the funds); given $e = (1, 1, \cdots, 1)^T \in R^N$, where $N$ is the number of funds, $v = A^T e$ gives the in-degree (of the benchmarks).[5] The most central nodes in accord to the in/out node-degree are the ones that have the highest number of links entering in or exiting from the node. Considering the nodes that represent the mutual funds, the out-degree corresponds to the diversification of the investments, while their in-degree is 0; considering a node representing a benchmark, the in-degree corresponds to the number of funds that invest in it (D'Arcangelis & Rotundo, 2015); their out-degree is zero. Standard analyses consider the hypothesis of Poisson, the exponential or power law distribution, for the applicability of models of network simulation and for using already existing results on resilience of networks (Newman, Barabàsi, & Watts, 2006).

On bipartite networks, the matrix $A$ is the basis for the one-mode projection: $B = (b_{ij}) = AA^T$ is the funds-funds (F-F) network: the value $b_{ij} \in \mathcal{N}$ is the amount of benchmarks which both fund $i$ and fund $j$ invest on. The matrix $C = (c_{ij}) = A^T A$ is the benchmarks-benchmarks (B-B) matrix: a value $c_{ij} \in \mathcal{N}$ is the amount of funds that refer at the same time to both the benchmarks $i$ and $j$ (D'Arcangelis & Rotundo, 2015). On both matrices, we repeat the analysis of the node-degree. On the one-mode projection networks (both F-F and B-B), the node-degree

[5] We remark that due to the 0 values in the diagonal blocks, the usage of either $A$ or $\widetilde{A}$ gives the same result, with a proper choice of the length of the vector $e$. The in-degree of the funds is 0 and the out-degree of the benchmarks is 0.

measures the overlap among the quantities represented by nodes (funds for the F-F and benchmarks for the B-B), and we deepen the study through other CM.

### 4.1. The one-mode projection networks: funds-funds and benchmarks-benchmarks matrices

This section deepens the analysis of the funds-funds and benchmarks-benchmarks network, as represented by the matrix $B$ and $C$, respectively. In the one-mode projection $B$, all the nodes correspond to pension funds, only. In the one-mode projection $C$, all the nodes correspond to the benchmarks, only. Nodes with high (low) out-degree have a high (low) overlap with other funds (benchmarks) in the selection of the benchmark (fund). By construction, $B$ and $C$ are symmetric matrices, so the in-degree equals the out-degree and can simply be called node-degree.

From the correlation of the node degree we may get hints on the presence of a rich-club or a general core-periphery network. The rich-club is a group of nodes with very high node degree and also highly interconnected among them. Each node in the rich-club has its group of *fans*, i.e. it is connected to nodes with a low number of connections.

The core-periphery model is characterized by a densely connected *core* linked to other nodes that do not necessarily show at once a sharp decrease in their number of connections, but the peripheral set of nodes is loosely connected to the core. It has been proved that the two structures respond quite differently to the propagation of shocks and fluctuations. Therefore, understanding the network topology increases the understanding of the robustness of the system and of its resilience to financial fluctuations (Cinelli, Ferraro, & Iovanella, 2017; Cinelli, Ferraro, Iovanella, & Rotundo, 2019; Newman et al., 2006).

The *assortativity* allows to discriminate between these two situations. In fact, technically, the assortativity is the correlation measure of the node degrees. It is not a centrality measure because it does not provide a ranking of nodes, but it is a quantity pertaining to the entire network. In the rich-club network the direct connection to low connected nodes causes a negative correlation, hence a so called negative assortativity. In the core-periphery, the smooth passage from the core to the external nodes allows room for a positive assortativity.

In order to have a better understanding of the network, we are going to examine the CM *betweenness* and *closeness*. The betweenness measures to which extent the node $i$ is "in between" two groups, i.e. how relevant it is for connecting two other nodes. This concept is expressed through the percentage of minimal paths connecting two nodes and passing through $i$ divided by the number of all the possible minimal paths:

$$\widetilde{C}^B(i) = \sum_{j<k} \frac{d_{jk}(i)}{d_{jk}}$$

where $d_{jk}$ is the number of the shortest paths between $j$ and $k$. $d_{jk}(i)$ is the number of the shortest paths between $j$ and $k$ that pass through $i$. The betweenness reaches its maximum for the nodes that provide the only connection among communities which would be separated otherwise. When groups are not so neatly distinct, the betweenness depends positively on the degree of the nodes, since more connections mean more possibilities to be on the shortest path among any two couples of other nodes.

The closeness of each node $i$ is calculated through $C_i = \frac{1}{\sum_j d_{ij}}$. The denominator contains the sum of all the distances of each node $j$ from node $i$. Therefore, the closeness centrality of a node is proportional to the inverse of the mean distance of the node from all the other. The node with the highest rank in accord to the closeness has the shortest distances from each of the others. This centrality measure differs from the node-degree, because it can happen that a well-connected node remains at the periphery of the entire network, far from a large part of the network.

In mutual funds, it has been shown that a high value of the betweenness actually evidences the funds that perform investments overlapping with the ones of different groups. Therefore, a high value of the betweenness actually may correspond to be not so central in terms of similarity to other funds (D'Arcangelis & Rotundo, 2015). This aspect will be checked for the field of pension funds, in our case study. In one mode projection networks, it is worth investigating the eventual belonging of pension funds to communities. In order to identify communities we rely on two different methods: the Louvain method for community detection (Blondel, Guillaume, Lambiotte, & Lefebvre, 2008) and the $k$-core and $k$-shells (Garas et al., 2012).

### 4.2. The Louvain method

The Louvain method for community detection takes its name from the main affiliation of the authors, who moved next to different working positions. The method follows a bottom-up approach: single nodes are grouped basing on the criterion of building communities that are as close as possible to a clique (a complete sub-network). On the funds-funds network it means that the communities gather the pension funds with the biggest overlap within the community, so putting together the pension funds that share the most similar decisions in the selection of benchmarks. From a theoretical point of view, the method proposes the optimization of the so called modularity, defined as:

$$Q = \frac{1}{2m}\sum_{ij}\left[A_{ij} - \frac{k_i k_j}{2m}\right]\delta\left(c_i, c_j\right)$$

w here $A$ is the adjacency matrix of the network; $k_i$ and $k_j$ are the sum of the weights of the edges attached to nodes $i$ and $j$, respectively. $2m$ is the sum of all the edge weights in the graph; $c_i$ and $c_j$ are the communities of the nodes; and $\delta$ is a simple delta function.

The algorithm unfolds a complete hierarchical community structure for the network. Each community is identified by a *module*. The size of the modules is driven by a parameter $\gamma$:

- $\gamma > 1$ detects "small modules"
- $0 \leqslant \gamma < 1$ detects "large modules"
- $\gamma = 1$ is the (default) "classic" modularity

A remarkable feature of the algorithm is to keep the computational complexity low. At the beginning, each node is a community. The node $i$ is assigned to the community of node $j$ if:

$\Delta Q > 0$

where $\Delta Q$ is defined as:

$$\Delta Q = \left[\frac{\Sigma_{in} + 2k_{i,in}}{2m} - \left(\frac{\Sigma_{tot} + k_i}{2m}\right)^2\right] - \left[\frac{\Sigma_{in}}{2m} - \left(\frac{\Sigma_{tot}}{2m}\right)^2 - \left(\frac{k_i}{2m}\right)^2\right] \quad (1)$$

When $i$ is moving to a community $\widetilde{C}$, $\Sigma_{in}$ is sum of all the weights of the links intra-community, $\Sigma_{tot}$ is the sum of all the weights of the links to nodes in $\widetilde{C}$, $k_i$ is the weighted degree of $i$, $k_{i,in}$ is the sum of the weights of the links between $i$ and other nodes in $\widetilde{C}$, and $m$ is the sum of the weights of all links in the network. The node $i$ is placed into the community that results in the greatest modularity increase among all the communities which $i$ is connected to. If $\Delta Q \leq 0$, the node $i$ remains in its original community. It is an iterative algorithm. The termination conditions of the first phase are that either all the nodes are allocated to communities or no modularity increase can occur. In the second phase of the algorithm, nodes inside the same community are substituted by super-nodes gathering all the information. Any links between nodes of the same community become self loops on the super node. Links from eventually more nodes in the same community to nodes in another one are represented by weighted edges between communities. The algorithm continues by applying again phase one and phase two till the desired degree of grouping communities. Of course, working with insulated nodes is not meaningful, since they remain out of any community or – viceversa – they can be attached to any community without any change of the information content. Disconnected networks instead will not gather the disconnected components into the same community, but inside each of them more communities can be evidenced.

### 4.3. k-core and k-shells

The method of $k$-core and $k$-shell is the basis of *onion networks*: the network is divided in layer ($k$-shells), and the nodes belonging to each of them are identified by the number of links that is necessary to disconnect them from the network. Recently, the methodology has been gathered to identify *onion* networks, where each $k$-shell is a layer (Hébert-Dufresne, Grochow, & Allard, 2016). Fig. 3 shows the following example:

- 1-shell:
  1. nodes with degree = 1 (red)
  2. nodes which degree becomes = 1 after the red ones are removed (black)
- 2-shell:
  1. nodes which degree = 2 (green) after 1-shell are removed.
  2. node which degree becomes = 2 (yellow) after the previous are removed.
- 3-shell: the blue ones

A node with degree $h$ cannot belong to a $k$-shell with $k > h$, but having degree $k$ does not guarantee to a node to be in the $k$-shell. Therefore: the degree is the maximum of the shells, but usually it does not correspond to the shell number. For instance, the black nodes have degree, respectively, 4 and 2, but they belong to the 1-shell 3.

When applied to the funds-funds matrix, this method gives a measure of the strength of the overlap of investments. In fact, if only the removal of one link is sufficient to remove a node (a fund), it means that the fund represented by the node only overlaps with only one other fund over all the possible selection of benchmarks. The higher the need of link removal, the stronger is the overlap among the funds. The nucleus with the highest $k$, i.e. the $k$-shell with the highest $k$ is the $k$-core.

From a technical point of view, the idea of $k$-shells arises from chemical shells. In chemistry and atomic physics, a $k$-shell describes the levels of electrons surrounding the nucleus of an atom; the ranking distribution is important for determining how the atom reacts chemically. In complex networks, the role of the atomic nucleus is played by the $k$-core, which is composed by the *most connected* nodes according to the following definition.

Let $S_k$ indicate the group of nodes that belong to the $k$-shell. Such nodes.

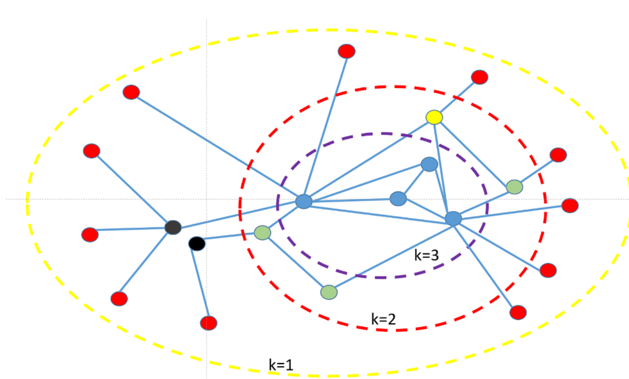


**Fig. 3.** Example of $k$-shells and $k$-core. The dashed lines serve to drive the eye and correspond to values of $k$. The most internal gives the shell with the highest $k$, i.e. the $k$-core. The nodes external to each dashed line remain disconnected when $k$ links are (recursively) removed.

1. have degree at least $k$ when all the nodes with degree at most $k$ are removed through a recursive procedure;
2. will be removed when all the nodes with degree at most $k + 1$ are removed.

The $k$-core is the last group that is removed, so leaving an empty set. Be aware that -in the recursive procedure- the removal of some nodes may reduce the degree of others that at the beginning have high degree. The reduction can be as severe as causing the removal of high degree nodes for low values of $k$. $k$-shells and $k$-cores are inducing a further centrality measure: the higher is $k$, the more central the role of the fund is. Of course, and differently from other centrality measures, most likely groups of nodes could have the same value, so there will be groups of nodes with the same centrality. The nucleus of the network is the set of the nodes in the highest shell. Therefore, the $k$-shell does not provide the same ranking as the degree. In fact, although nodes with degree $h$ cannot belong to $k$-shells with $k > h$, they will not be automatically assigned to the shell with $k = h$, because the recursive procedure of node removal may dramatically decrease the degree of other nodes.

## 5. Results

### 5.1. The 0/1 funds-benchmarks network

We draw in Fig. 4 the bipartite 0/1 funds-benchmarks matrix. The drawing emphasizes the bipartite structure. The bottom layer reports all the funds (1–61). The higher levels report the benchmarks (62–105). The higher the level, the more connected the benchmark. The most connected are: Msci World (43 connections), Euro Stoxx (27 connections), JP Morgan Euro Government Bond (16 connections) and MSCI North America (15 connections).

Fig. 5 reports the histogram of the out-degree $u$ and in-degree $v$, respectively. The out-degree represents the diversification of mutual funds investments on the benchmarks. The in-degree reports the integration of benchmarks in pension funds investments.

Following the standard analyses, the hypothesis of Poisson, exponential and power law distributions have to be tested. Clearly, the histograms of $u$ and $v$ do not represent a gaussian distribution, as further confirmed by the Q-Q plot and the Jarque-Bera test.[6] While the distribution of $u$ does not fit well any specific distribution, the histogram of $v$ is best described by a power law function $p(x) \sim ax^{-b}$, with $a = 13.00(7.34, 18.66)$, and $b = 2.47(2.09, 2.84)$, $R^2 = 0.99$.

Within this context, we do not deepen the analysis on stochastic models for network formation. Our aim is to inquire the pension funds' investment styles. From the histogram in Fig. 5 (left panel: out-degree), we point out that most pension funds declare 2–4 benchmarks, while only two funds, Gommaplastica-Dinamico and Fondapi-Crescita declare many benchmarks, (9 and 7 respectively). Differently from mutual funds, generally declaring a single benchmark, pension funds used to declare a set of benchmarks. This is a typical feature of pension funds investments' management as they used to give the mandates to external managers based on the benchmarks.

The results on the benchmarks (Fig. 5, right panel: in-degree) show that the most popular benchmark is the MSCI World (43 funds have declared it), the next is the Euro Stoxx (27 funds have declared it). This result was expected as the MSCI World is a broad global equity benchmark including equities from 23 developed markets countries while Euro Stoxx is a broad yet liquid subset of the Stoxx Europe 600 index, representing large, mid and small capitalization companies of 11 Eurozone countries.

[6] The Jarque-Bera statistic is: $JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$, where $n$ is the number of observations (or degrees of freedom), $S$ is the asymmetry of the dataset, $K$ is the kurtosis.

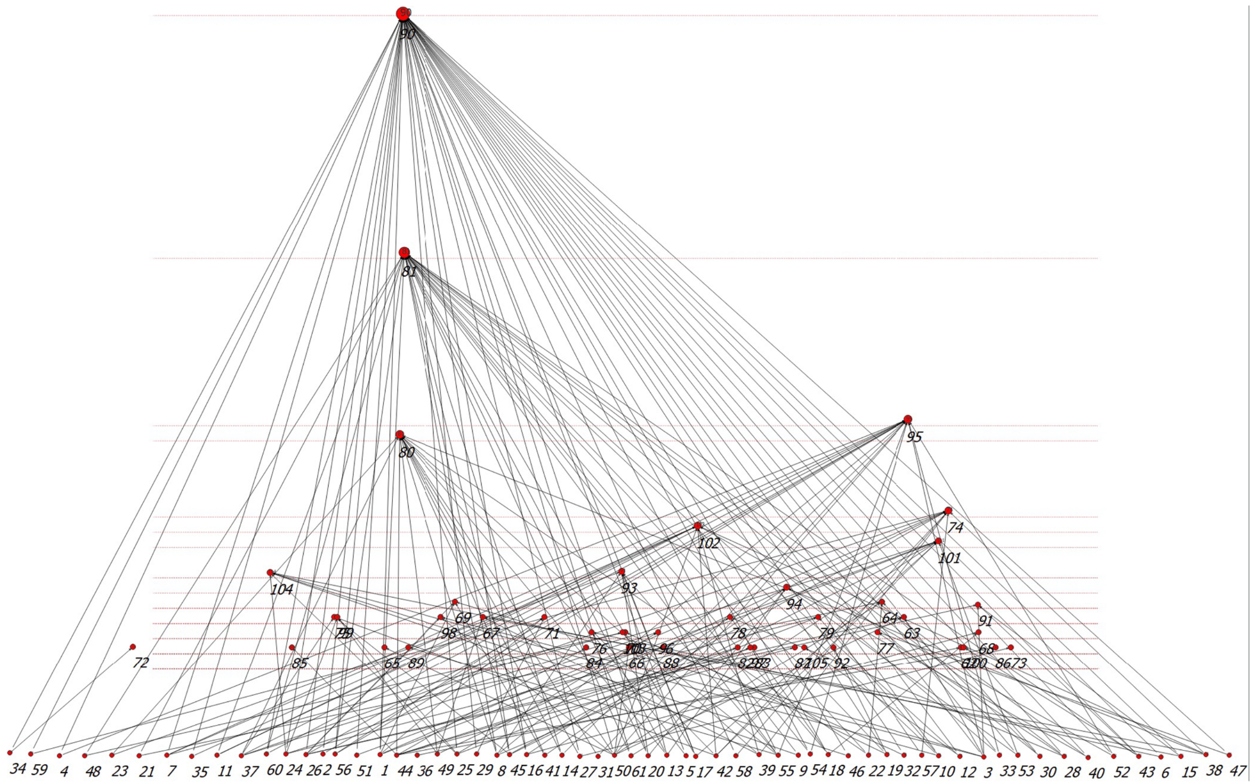

**Fig. 4.** Bipartite 0/1 funds-benchmarks network. The bottom layer reports all the funds (1–61). The higher levels report the benchmarks (62–105).

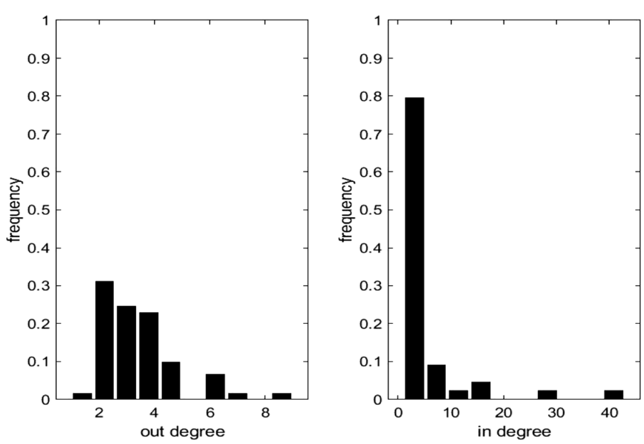


**Fig. 5.** Histograms of the node degrees of the 0/1 funds-benchmarks network: out-degree (left panel) and in-degree (right panel).

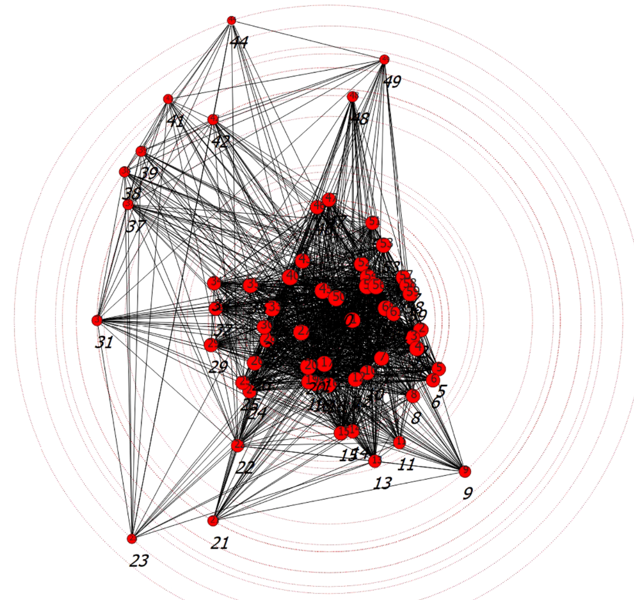

**Fig. 6.** The 0/1 funds-funds network.

### *5.2. The 0/1 funds-funds network*

This section deepens the analysis of the funds-funds network, represented by the matrix $B$, keeping only the 0/1 structure. In this one-mode projection, all the nodes correspond to pension funds, only. Nodes with high (low) out-degree have a high (low) overlap with other funds in the selection of benchmarks. By construction, $B$ is a symmetric matrix, so the in-degree equals the out-degree and can simply be called node-degree. The 0/1 funds-funds matrix is drawn in Fig. 6.

Fig. 7 shows the histogram of the node degree of the funds-funds network, describing the overlap among pension funds in their declaration of the benchmark's choice. In the figure's caption we also report the main statistics. The mode value (that is equal to 50) means that 50 pairs of pension funds have an overlap of their investments on at least one benchmark. The MSCI world and Euro Stoxx benchmarks contribute to a large extent of this value.

The most overlapped fund is Fonchim-Crescita (56 connections); the next most connected one is Previmoda-Rubino Azionario (55 connections). The fund Azimut previdenza-Crescita has 0 overlap since it does not declare a traditional financial benchmark but pursues the objective, over a long-term horizon, of a positive absolute return at least equal to

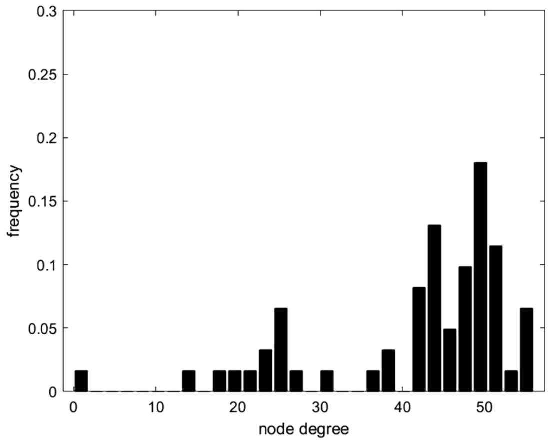


**Fig. 7.** Histogram of the node degree of the 0/1 funds-funds network. Main statistics: max = 54, min = 0, mean = 41.86, mode = 50.

the 75% ISTAT FOI index.[7] The ISTAT index to which Azimut previdenza refers to is the annual percentage variation of FOI index plus a 2.5% yield. In the prospectus, the management declares that to achieve the above-mentioned objective, it is oriented towards equity financial instruments. The company has however the power to reduce significantly (eventually to zero) the equity component in a dynamic and flexible manner. Actually, the fund Azimut previdenza has a flexible investment style: it has invested 86.44% of portfolio assets in stocks, 13.55% in cash and deposits and 0.01% in bonds in 2017.

Making a step further on the correlation among the node-degrees, we note that the assortativity has a positive value, 0.1578, giving room for the hypothesis of a core-periphery model (see Fig. 6). The skewness indicates the presence of a large core. This induces us to investigate the presence of clustering.

- *Other centrality measures on the 0/1 funds-funds network: betweenness and closeness*

In Fig. 8, we show the results of the betweenness (left panel) and closeness (right panel) on the 0/1 funds-funds matrix. We recall that the betweenness is maximal for units (bridging) groups that would be otherwise separated, while the closeness represents the distance of a node from the others. The results on the betweenness are in line with the results on the node-degree: most pension funds have low value of the betweenness, being actually merged in a dense network. Of course, the disconnected node has betweenness 0. The node with the highest value of the betweenness is Gommaplastica-Dinamico, probably due to the high number (9) of benchmarks declared by this fund. As regards to the closeness, the skewness is well in line with the previous result, confirming a well connected network with short distances.

- *Louvain method on the 0/1 funds-funds network*

The Louvain method is used to detect communities in networks. It has been run with the $\gamma$ parameter of modularity equal to 0.9, that takes into account large communities. The nodes are classified in 3 communities (group 1, 2 and 3), apart from the disconnected nodes, that remains insulated (the fund Azimut previdenza – crescita). In Table 6 and Fig. 9 we show the Louvain groups by pension fund category and type of asset manager. Specifically, Table 6 shows the proportion of contractual and open pension funds in each community. We can observe that group 2 includes the most contractual pension funds (63.2%), while the open pension funds are mainly in group 3 (52.4%) and group 1 (28.6%).

Fig. 9 shows the percentage composition of each community by pension funds category. Groups 1 and 2 mostly contain open pension funds, 85% and 82% respectively. While, in group 2 only 37% are open funds.

- *k-shell of the 0/1 funds-funds network*

The vast majority of nodes (43 over 61) belongs to the inner shell ($k$-core with $k = 43$), while only approximately $1/3(=18/61)$ is out of it (Fig. 10). Therefore, with 1605 links over the max of 1849 in the core, the network shows a dense structure of connections.

Funds in group 1 in accord to the Louvain method are all included in the $k$-core, apart the fund Vittoria formula lavoro. Only 33% of the funds in group 2 Louvain are in the $k$-core. Group 3 includes most nodes in the core, apart from two (Eurorisparmio azionario Europa e Fondo pensione aperto BIM vita equity).

### 5.3. The 0/1 benchmarks-benchmarks network

The analysis in this section relies on the benchmarks-benchmarks network, represented by the matrix C, keeping only the 0/1 structure. In the one-mode projection $C$, all the nodes correspond to benchmark, only. Nodes with high (low) out-degree have a high (low) overlap with other funds in the selection of benchmarks. The 0/1 benchmarks-benchmarks network is drawn in Fig. 11.

The network has 44 nodes. The following outcomes are noteworthy:

- 75% ISTAT +2.5% is not connected to any other benchmark, i.e. the pension fund declaring this benchmark does not declare other benchmarks in common with any other listed funds.
- MSCI world: overlapped with 34 other benchmarks.
- Euro Stoxx: overlapped with 26 other benchmarks.
- MSCI North America: overlapped with 25 other benchmarks.

MSCI world and Euro Stoxx are in line with their high in-degree value in the funds-benchmarks network.

The distribution of the node degree (Fig. 12) is clearly not a power law and is skewed, showing a prevalence of low degrees nodes, that means that benchmarks are mostly bought by a few funds.

The negative value of the assortativity, $-0.3563$, suggests the presence of a rich club structure, with hubs connected to low degree nodes.

- *Other centrality measures on the 0/1 benchmarks-benchmarks network: betweenness and closeness*

Fig. 13 (left panel) shows the betweenness of the 0/1 benchmarks-benchmarks network. Many nodes have betweenness close to 0. This implies that the groups are not well defined, and that the role of nodes "bridging" communities is limited.

Fig. 13 (right panel) shows the closeness of the 0/1 benchmarks-benchmarks network. Apart of the insulated benchmark 75% ISTAT + 2.5% reporting closeness 0, most of the nodes have a short closeness.[8] This means that there are no long chains in the network. Recalling the meaning of the nodes, the absence of long chains implies that many

[7] The "FOI" index (indice dei prezzi al consumo per Famiglie di Operai e Impiegati) is the index of consumer price for blue- and white-collar households published by the Italian National Institute of Statistics (ISTAT).

[8] We recall that small world hypothesis holds when the diameter of a network is less than logarithm of the number of nodes ($log(N) = log(44) = 3.78$); and it is supported by a high level of the clustering coefficient, which is the number of closed triplets over all the possible ones (both open and closed) Varela Cabo and Rotundo (2016). The prevalence of short distances supports the small world hypothesis. For further cross-check we calculated the diameter ($=3$) and the clustering coefficient ($=0.79$).

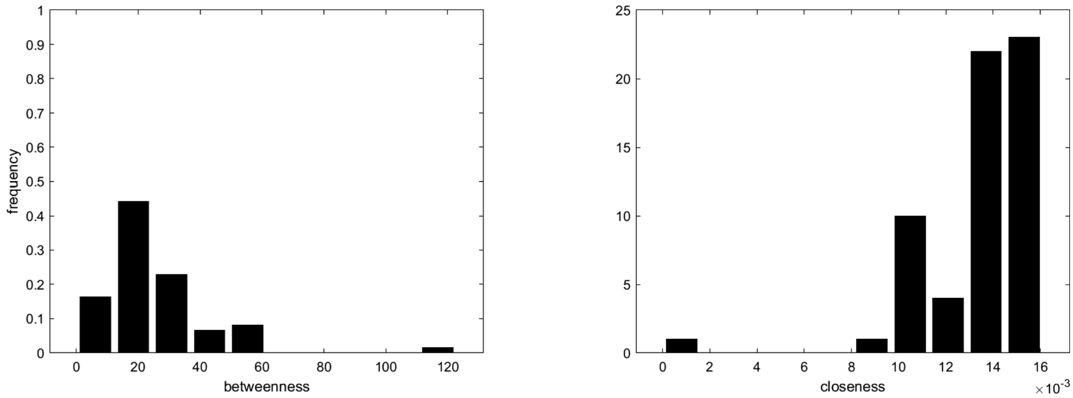


**Fig. 8.** Histogram of the betweenness (left panel) and closeness (right panel) on the 0/1 funds-funds network.

**Table 6**
Pension funds communities from the Louvain method on the 0/1 funds-funds network. Proportion of contractual and open pension funds in each community.

| Pension fund category | Asset manager | Group 1 | Group 2 | Group 3 | NA | Total |
|---|---|---|---|---|---|---|
| Contractual | All | 10.5% | 63.2% | 26.3% | – | 100.0% |
| | – Asset management companies | 5.3% | 42.1% | 21.1% | – | 68.4% |
| | – Other[2] | 5.3% | 21.1% | 5.3% | – | 31.6% |
| Open | All | 28.6% | 16.7% | 52.4% | 2.4% | 100.0% |
| | – Insurance companies | 23.8% | 11.9% | 40.5% | 2.4% | 78.6% |
| | – Asset management companies | 4.8% | 4.8% | 11.9% | – | 21.4% |

[2] Stock brokerage companies and private banking.

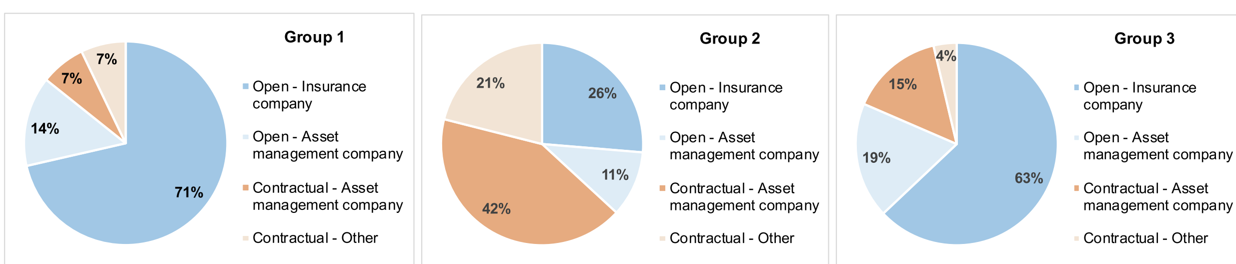


**Fig. 9.** Pension funds communities from the Louvain method on the 0/1 funds-funds network. Percentage composition of each community by pension funds category.

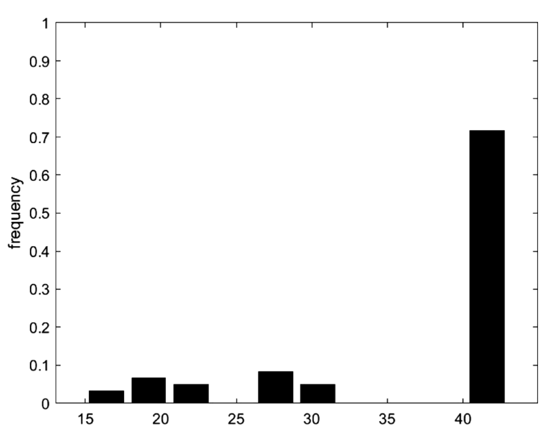


**Fig. 10.** Histogram of the shells on the funds-funds network.

couples of funds declare the same benchmarks; therefore, they are quite well connected each to the other. This result is mainly due to MSCI world and Euro Stoxx indexes that have high node degree.

- *The Louvain on the 0/1 benchmarks-benchmarks network*

The method proceeds gathering together nodes with similar features, and it stops when the desired maximum number of communities is reached. We firstly classify the nodes in three communities, obtained by setting $\gamma = 1$ (see Table 9, column 4, shown in the Appendix). We observe that the detection of the nodes belonging to the same community is not perfectly correlated with the node degree, i.e. nodes with high degree are distributed in different communities. This happens with several communities. Further, classifying the nodes in four communities ($\gamma = 1.1$, Table 9, column 5), we note that all the equity benchmarks belong to the same community (group 1). Conversely, when the benchmarks are grouped in three communities, the equity benchmarks are split in two different groups characterized by high geographical diversification and mutually exclusive. The most popular are MSCI Europe and MSCI North America in the first community, the widest with 26 benchmarks and the most connected, and Eurostoxx and MSCI World in the second community.

- *The k-shell on the 0/1 benchmarks-benchmarks network*

The *k*-shell result contains a restricted core (9 benchmarks, shell number 9), surrounded by the large neighborhood of the 7-shell (14 benchmarks). There is an overlap with the results of the Louvain method with four communities (parameter $\gamma = 0.6$):

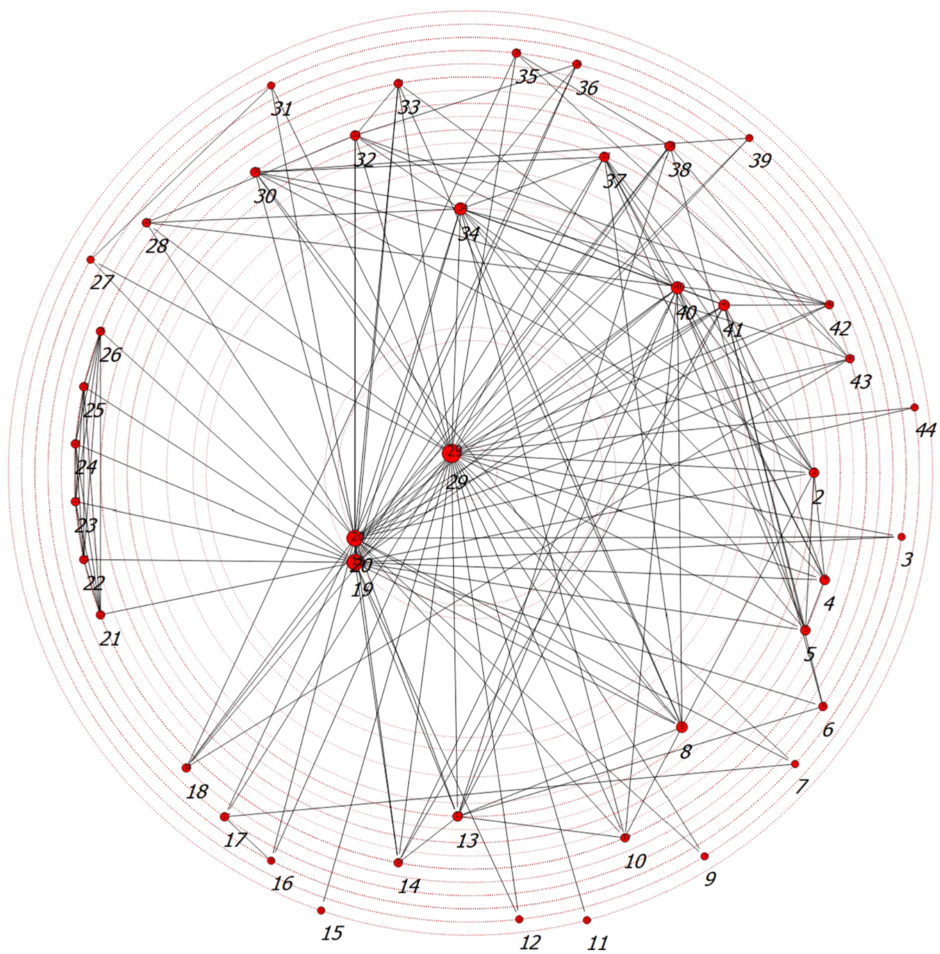


**Fig. 11.** The 0/1 benchmarks-benchmarks network.

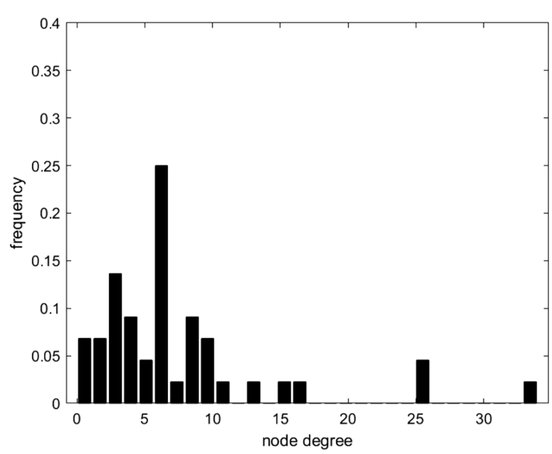


**Fig. 12.** Histogram of the node degree on the 0/1 benchmarks-benchmarks network. Main statistics: max = 34, min = 0, mean = 7.45, mode = 6.

- The $k$-shell with $k = 4$ contains only elements of the first group of Louvain method.
- The $k$-shell with $k = 7$ contains all the elements of the forth group of Louvain method.
- The $k$-shell with $k = 9$ contains more than 50% of the elements of the second group of Louvain method.

From the analysis we note that the $k$-shell and Louvain methods provide different results. The communities that are detected by both the methods surely have stronger connections compared with the others. For instance, the most of the Bofa Merrill Lynch and all the Ecpi (Social Responsibility Investments) for shell 7 and all the MSCI and almost all the Barclays Capital for shell 9.

### 5.4. The weighted funds-benchmarks network

Fig. 14 shows the histogram of the values inside the adjacency matrix, that is all the weights of all the links of the network. From the figure it clearly emerges that many funds declare a few benchmarks (low values on the right side of the figure) and very few funds declare many benchmarks (high values on the left side of the figure). We can note that the histogram shows a nearly monotonic decrease with no relevant peaks. Both the exponential and the power low regression give an $R^2$ lower than the other regression that we are showing on the node degree; thus, we do not consider the matter further.

Fig. 15 considers the in-degree of the weighted funds-benchmarks network and shows clearly that the vast majority of the non-zero links still have small weight.

The best fit is given by the exponential decay $p(x) \sim e^{-\alpha x}$ with $\alpha = 3.37$, which implies that only a few funds have high weights, revealing a concentration of their investments on one-two benchmarks, only. When the weights are considered, the out-degree in the funds-benchmarks network gives result 1 for all the funds: in fact the dataset reports the 100% of the investments of each fund on the benchmark.

### 5.5. The weighted funds-funds network

Fig. 16 shows the weighted funds-funds network. The weights are well centered around the mean. The most overlapped funds are Insieme

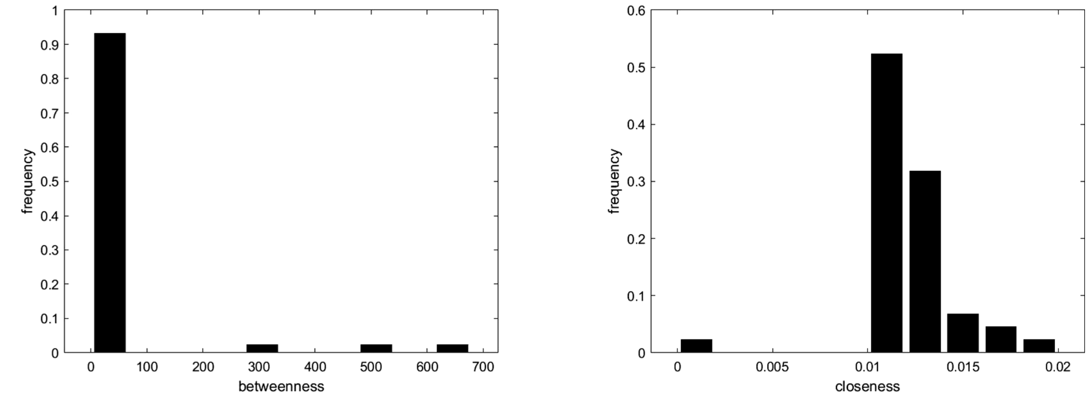


**Fig. 13.** Histogram of the betweenness (left panel) and closeness (right panel) of the benchmarks-benchmarks network.

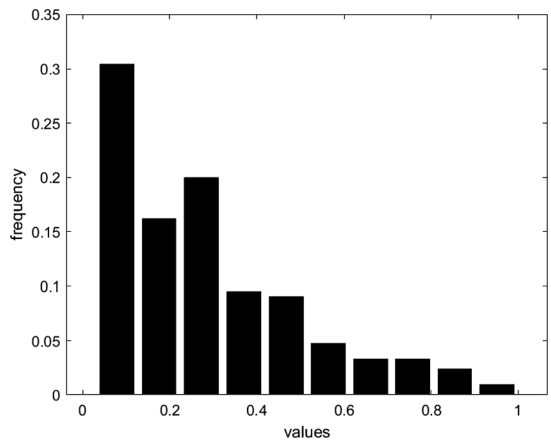


**Fig. 14.** Histogram of the values in the weighted funds-benchmarks network. Main statistics: max = 1, min = 0.3, mean = 0.30, mode = 0.05.

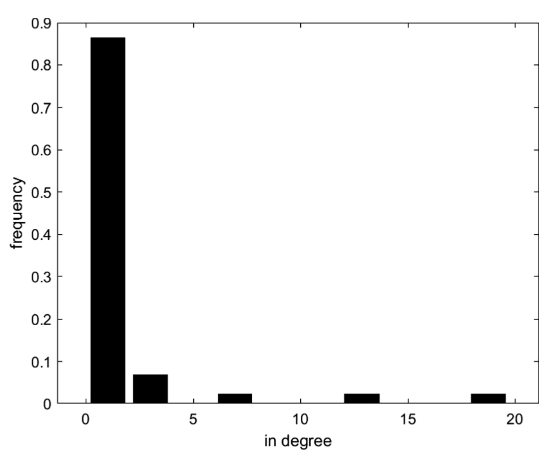


**Fig. 15.** Histogram of the weighted in-degree of the weighted funds-benchmarks network. Main statistics: max = 19.74, min of the non-zero values = 0.04, mean = 1.38, mode = 0.1.

– linea azionaria and Allianz Previdenza – linea azionaria. The weights play an important role to detect the most connected funds providing results different from the 0/1 case (Fonchim – Crescita and Previmoda – Rubino Azionario). The fund Azimut Previdenza – Crescita is not overlapped with anyone else. The assortativity is 0.1578, positive, so the connections among nodes with similar degrees have a light prevalence in the set of possible links. The positiveness of the assortativity excludes the rich club as eventual model for this network.

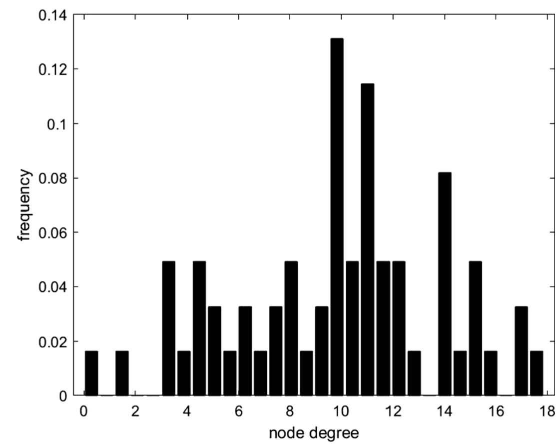


**Fig. 16.** Histogram of the node degree in the weighted funds-funds network. Main statistics: max = 17.87, min = 0, mean = 9.85, mode = 14.17.

- *Other centrality measures on the weighted funds-funds network: betweenness and closeness*

Fig. 17 shows the histogram of the betweenness (left panel) and closeness (right panel). The values of the betweenness are quite similar to the case 0/1, with analogous remarks. On the closeness, at a first sight, the behavior is quite different from the 0/1 case, where most of the values were between 0.01 and 0.02. In the weighted matrix, the closeness ranges in [0.1, 0.3] with a growing tendency. However, the range is so small that the nodes can be considered quite well connected in both cases. Gommaplastica – Dinamico remains the node with the highest betweenness respect to the 0/1 case, so confirming the relevance of its high number of connections.

- *Louvain method for detecting communities on the weighted funds-funds network*

Running again the algorithm on the weighted network, most of the results already outlined for the 0/1 case are confirmed, showing the high relevance of the topology of the network for the detection of communities and overlaps of investment styles. The few differences that emerge from the Louvain community detection in the weighted network can be summed up in a lower presence of open pension funds managed by insurance companies in Group 1, and in their bigger presence in Group 3. Group 2 most contains the contractual pension funds, as in the 0/1 case. Of course, the disconnected node (Azimut Previdenza – Crescita) remains insulated.

In Table 7 we report the proportion of contractual and open pension funds in each community. As already observed in the 0/1 case (see Table 6), group 2 contains the most contractual pension funds (52.6%),

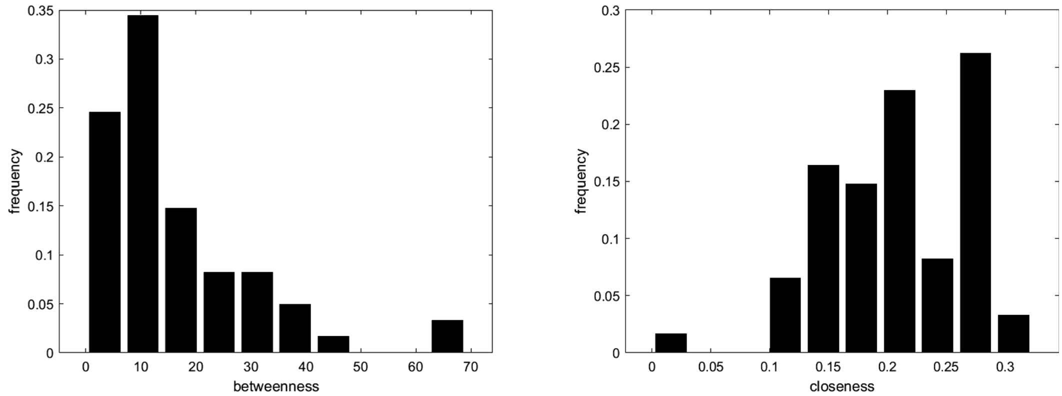


**Fig. 17.** Histogram of the betweenness (left panel) and closeness (right panel) in the weighted funds-funds network.

**Table 7**
Pension funds communities from the Louvain method on the weighted funds-funds network. Proportion of contractual and open pension funds in each community.

| Pension fund category | Asset manager | Group 1 | Group 2 | Group 3 | NA | Total |
|---|---|---|---|---|---|---|
| Contractual | All | 42.1% | 52.6% | 5.3% | – | 100.0% |
| | – Asset management companies | 15.8% | 47.4% | 5.3% | – | 68.4% |
| | – Other [3] | 26.3% | 5.3% | 0.0% | – | 31.6% |
| Open | All | 33.3% | 16.7% | 47.6% | 2.4% | 100.0% |
| | – Insurance companies | 28.6% | 11.9% | 35.7% | 2.4% | 78.6% |
| | – Asset management companies | 4.8% | 4.8% | 11.9% | – | 21.4% |

[3] Stock brokerage companies and private banking.

while the open pension funds are mainly in group 3 (47.6%).

In Fig. 18 we show the percentage composition of each community by pension funds category. Group 3 includes almost entirely open pension funds (95%), their percentage decreases to 64% in group 1 and 41% in group 2.

- *k-shell on the weighted funds-funds network*

Due to the rescaling of the weights, the fund Insieme – linea bilanciata is the most popular, being in the shell 51. Most of the elements belong to the most external shells. This is in favor of the presence of a very limited number of hubs, surrounded by many low connected units. Fig. 19 shows the histogram of the shells distribution in the weighted funds-funds network.

The results match the ones of the Louvain method most on the intermediate shells. The internal core is split into more shells.

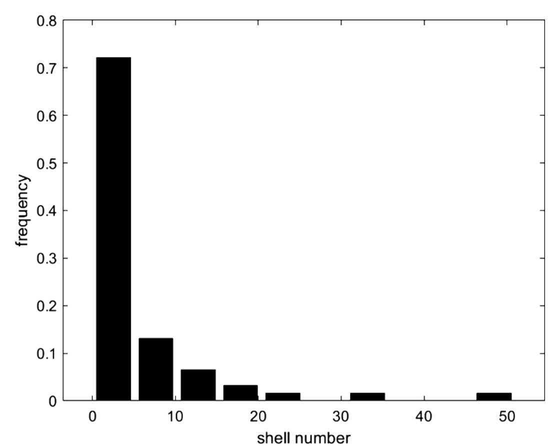


**Fig. 19.** Histogram of the shells in the weighted funds-funds network.

### 5.6. *The weighted benchmarks-benchmarks network*

The funds that do not overlap with the others in the benchmarks-benchmarks 0/1 network still do not overlap with anyone else. In fact, the weights are considered only for the nodes that already exist. The histogram of the overlap in the weighted benchmarks-benchmarks network is reported in Fig. 20. The most popular benchmarks are MSCI World and Euro Stoxx. Both are broad global equity benchmarks representative of capitalization companies worldwide and in the Eurozone countries. The next group insulated from the bulk are MSCI North America, JP Morgan Euro Government Bond and Bofa Merrill Lynch Pan-Europe Governative.

They have the largest overlap, this means that many funds invest in

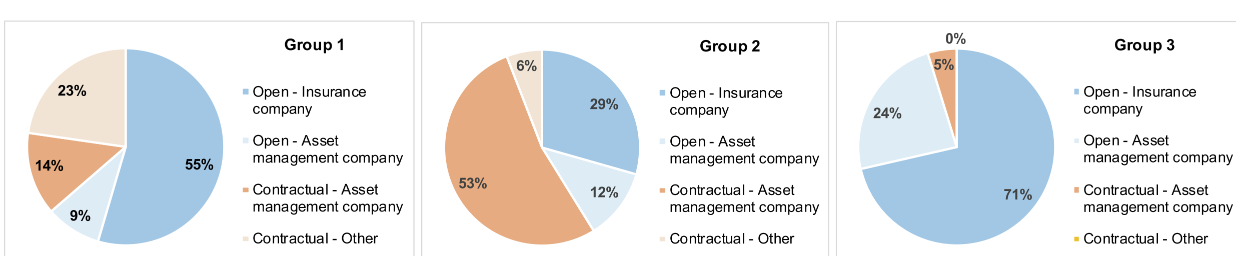


**Fig. 18.** Pension funds communities from the Louvain method on the weighted funds-funds network. Percentage composition of each community by pension funds category.

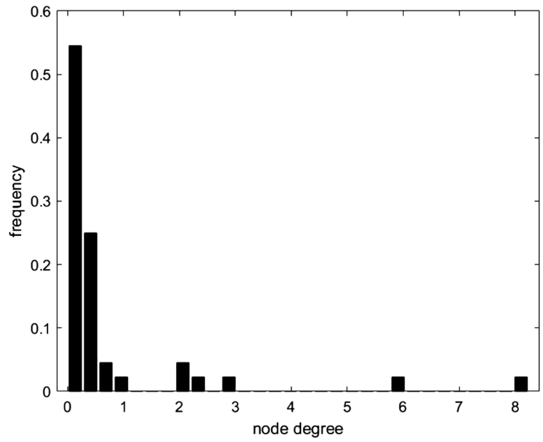


**Fig. 20.** Histogram of the node degree in the weighted benchmarks-benchmarks network. Main statistics: max = 8.25, mean = 0.52, mode = min = 0.

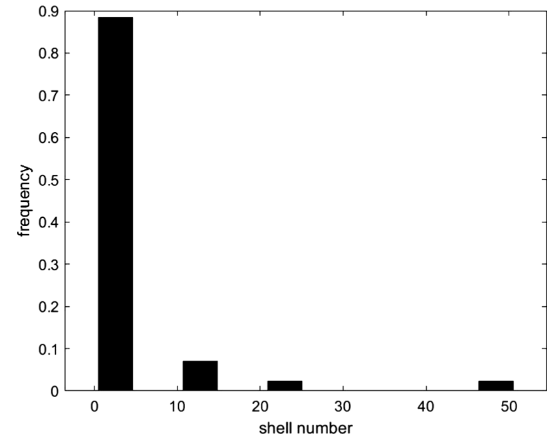


**Fig. 22.** Histogram of the shells in the weighted benchmarks-benchmarks network.

each of them. The assortativity is −0.3563, negative, so the network is disassortative, giving room for the hypothesis of a rich club network.

- *Other centrality measures on the weighted benchmarks-benchmarks network: betweenness and closeness.*

Fig. 21 shows the betweenness (left panel) and closeness (right panel) in the weighted benchmarks-benchmarks network. From the left panel, we observe that there are many nodes with betweenness 0. This is confirming that there are not so clearly distinct communities, so no "bridges" among them, apart from the most popular, which are Barclays Capital US Credit Index, MSCI World and MSCI Emerging Markets. The histogram in the right panel of Fig. 21 representing the closeness, shows a skewness. This is in line of the findings with the betweenness: the network shows quite homogenous connections and short distances.

- *Louvain on the weighted benchmarks-benchmarks network*

The method applied to the weighted benchmarks-benchmarks network detects communities that do not clearly evidence common characteristics from a benchmarks perspective. In this view the Louvain method does not show a satisfactory discriminatory power.

- *k-core on the weighted benchmarks-benchmarks network*

Fig. 22 shows the histogram of the shells distribution in the weighted benchmarks-benchmarks network. The core is even more restricted, and most nodes are in the outer shell. This means that they are loosely connected. This is in line with the most popular declared benchmarks. In the weighted case the actual node degree is normalized by using the weights (Garas et al., 2012). On the benchmarks-benchmarks network, the core is given by the Euro Stoxx, only. The closest shell contains only JP Morgan Euro Government Bond, followed by the shell with MSCI Emerging Markets and MSCI North America. This results evidence well the most popular and connected benchmarks. The remaining benchmarks belong to lower level shells.

## 6. Conclusions

In this work, we examine the Italian pension funds and their exposures to the self-declared benchmarks. Before starting the analysis, we performed a pruning identifying and merging the benchmarks with correlations above 90%. It is worth noting that some funds were declaring a few high correlated benchmarks. At first, we build the 0/1 funds-benchmarks bipartite network and its one-mode projections (funds-funds and benchmarks-benchmarks). This analysis serves to outline the role of network topology for determining the relevance of benchmarks and the similarities of investment styles of Italian pension funds without considering the percentages invested. The allocation of investments in different geographical areas appears with evidence. The centrality measures properly point out the most representative benchmarks. The network shows short distances, where each node is overlapped to the others in a very few steps. The overlaps are mainly due to the MSCI World and to Euro Stoxx, both represented by nodes with high degree.

The funds-funds network shows high overlaps and most funds are merged in a dense network. The addition of weights shows concentration on the low percentages on most benchmarks, apart from the most popular ones. The detection of communities through their 0/1 connection structure and the Louvain method partially gets the information

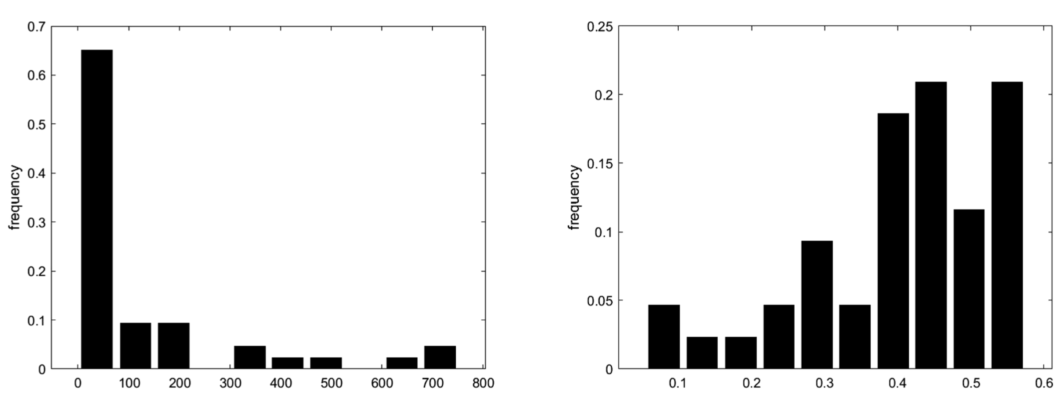


**Fig. 21.** Histogram of the betweenness (left panel) and closeness (right panel) in the weighted benchmarks-benchmarks network.

of the category of funds. This implies that – beside the formal category – there are similarities of styles. Most contractual pension funds are gathered in a specific community. The *k*-core gives good agreements, with some discrepancies. The overall funds-funds network is assortative, while the benchmarks-benchmarks network is disassortative, with clear difference among the most central and the others. However, when the weights are considered, the communities detected through the Louvain method seem not to capture well the funds features. Still, the *k*-core and the highest *k*-shell properly evidence the most popular benchmarks. The betweenness and the closeness are the quantities less changed by the introduction of the weights. In conclusion, the network structure – without considering information about the weights – contains already sufficient information for detecting similarities in investments styles of the Italian pension funds. This result gives new insights in the structure of the pension funds and can be used as the basis for future work.

## Appendix A

See Tables 8 and 9.

**Table 8**
List of the original and reference benchmarks.

| Original benchmark | Reference benchmark |
|---|---|
| Comit Performance | Comit Global |
| MSCI All Country World | MSCI North America |
| MSCI All Country World Free | MSCI North America |
| Dow Jones Sustainability World | MSCI North America |
| MSCI World ex Europe | MSCI North America |
| MSCI World ex Japan | MSCI North America |
| Stoxx Usa 900 | MSCI North America |
| ECPI World ESG Equity | MSCI North America |
| E-Capital Ethical Emu | Euro Stoxx |
| Euro Stoxx 50 | Euro Stoxx |
| MSCI All Country Europe | Euro Stoxx |
| MSCI Emu | Euro Stoxx |
| Stoxx Europe 600 | Euro Stoxx |
| FTSE All World Series All World Developed | MSCI World |
| MSCI World ex Emu | MSCI World |
| S&P 500 | MSCI World |
| FTSE Italia All share | FTSE MIB |
| MSCI Italy | FTSE MIB |
| MSCI Far East Pacific | MSCI Pacific |

**Table 9**
Benchmarks communities from the Louvain method on the 0/1 benchmarks-benchmarks matrix, by geographic area and asset class. L3: Louvain based on three communities, L4: Louvain based on four communities.

| Benchmark | Geographic area | Asset class | L3 | L4 |
|---|---|---|---|---|
| 75% ISTAT + 2.5% | Italy | Economic | 0 | 0 |
| Barclays Capital Pan European Aggregate Credit | Europe | Interest rate | 1 | 1 |
| Ecpi Global developed Esg Best in class equity | Global | Equity | 1 | 1 |
| MSCI Emerging Markets | Emerging Markets | Equity | 1 | 1 |
| MSCI Europe | Europe | Equity | 1 | 1 |
| MSCI North America | World | Equity | 1 | 1 |
| Barclays Capital US Credit Index | US | Interest rate | 1 | 1 |
| Barclays Euro Govt. Inflation linked All Markets ex Greece | Eurozone ex Greece | Interest rate | 1 | 1 |
| Bofa Merrill Lynch Italy Treasury Bill | Italy | Interest rate | 1 | 1 |
| Bofa Merrill Lynch Us Treasury Note | US | Interest rate | 1 | 1 |
| Ecpi Emu Governance Government bond | Eurozone | Interest rate | 1 | 1 |
| Ecpi Emu Governance Government bond Inflation linked bond | Eurozone | Interest rate | 1 | 1 |
| Euro Stoxx | Eurozone | Equity | 2 | 1 |
| FTSE 100 | UK | Equity | 2 | 1 |
| FTSE Mib | Italy | Equity | 2 | 1 |
| MSCI Pacific | Pacific | Equity | 2 | 1 |
| MSCI World | World | Equity | 2 | 1 |
| Barclays Capital Euro Inflation Linked Government Bond | Eurozone | Interest rate | 2 | 1 |
| Bofa Merrill Lynch Emu Corporate | Eurozone | Interest rate | 2 | 1 |
| Euromts Eonia Investable | Eurozone | Interest rate | 2 | 1 |
| FTSE Mts Eurozone Government Bond Investment Grade | Eurozone | Interest rate | 2 | 1 |
| JP Morgan Emu Cash | Eurozone | Interest rate | 2 | 1 |
| JP Morgan Emu Government Bond Investment Grade | Eurozone | Interest rate | 2 | 1 |
| JP Morgan GBI broad traded | Global | Interest rate | 2 | 1 |
| JP Morgan Global Government Bond ex Emu | Global ex Eurozone | Interest rate | 2 | 1 |
| JP Morgan Europe | Europe | Interest rate | 3 | 1 |
| Mts Italy BOT ex-Bank of Italy | Italy | Interest rate | 3 | 1 |
| Bofa Merrill Lynch Emu Direct Government Bond | Eurozone | Interest rate | 2 | 2 |
| Bofa Merrill Lynch Euro Treasury Bill | Eurozone | Interest rate | 2 | 2 |

(*continued on next page*)

**Table 9** (*continued*)

| Benchmark | Geographic area | Asset class | L3 | L4 |
|---|---|---|---|---|
| Bofa Merrill Lynch Global Government Bond ex Japan | Global ex Japan | Interest rate | 2 | 2 |
| Citigroup Euro Government Bond | Eurozone | Interest rate | 2 | 2 |
| Citigroup Eurobig | Eurozone | Interest rate | 2 | 2 |
| Citigroup World Government Bond Non Euro | World ex Eurozone | Interest rate | 2 | 2 |
| Barclays Capital Euro Treasury | Eurozone | Interest rate | 2 | 3 |
| JP Morgan Euro Government Bond | Eurozone | Interest rate | 2 | 3 |
| Libid | Eurozone | Interest rate | 2 | 3 |
| JP Morgan US Government Bond | US | Interest rate | 3 | 3 |
| Bofa Merrill Lynch Pan-Europe Governative | Europe | Interest rate | 1 | 4 |
| Ecpi Global developed Esg corporate ex financials bond | Global | Interest rate | 1 | 4 |
| Ecpi Global developed Esg corporate financials bond | Global | Interest rate | 1 | 4 |
| Ecpi Global developed ex Emu Governance Government bond | Global ex Eurozone | Interest rate | 1 | 4 |
| Bofa Merrill Lynch Emu | Eurozone | Interest rate | 2 | 4 |
| Mts Tasso Fisso Breve Termine | Italy | Interest rate | 2 | 4 |
| Comit Global | Italy | Equity | 3 | 4 |

## Appendix B. Supplementary material

Supplementary data associated with this article can be found, in the online version, at https://doi.org/10.1016/j.jbusres.2019.10.071.

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