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The simple analytics of optimal growth with migration

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Abstract This paper investigates the economic consequences of migration in the Ramsey-type dynamic optimizing context. In contrast to Hazari and Agro (J Econ Dyn Control 28:141–151, 2003) conclusions, we show that migration unambiguously reduces the per-capita domestic consumption growth, whereas necessarily raises the long-run per-capita consumption of domestic residents when production is "sufficiently" reactive to capital changes. Our findings are consistent with several empirical studies and simulation analyses, suggesting that changes in technological adjustment in response to migrants inflows may take some years to translate into productivity, generating some crowding out effects. The gains for natives are likely to materialize in the long run when the specialization of natives adjusts, firms invest in capital and adopt appropriate technologies.

Keywords Domestic consumption · Growth · Migration

JEL Classification F2 · O4

1 Introduction

In recent years, policy-makers of many countries are increasingly concerned about growing migration pressure from developing countries. Notably, the rapid growth of population,

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changes in geo-political structure and the wide inequality in income have led to a rapid increase in the number of migrants from emerging economies to advanced ones (see e.g. Hanson 2006, 2010; Hanson and McIntosh 2010). In some cases migrant-receiving countries aim at maintaining a balance between economic needs, triggered by an ageing domestic workforce, and political commitments which typically result in a high restrictive migration policy. As a consequence, the need of understanding this mounting phenomenon has gained a remarkable revival of interest in questions concerning the migrants flows determinants, economic causes and features, and effects on welfare (among others see Borjas 2003; Hanson 2009; Peri 2009). In addressing these issues, the literature has examined, in particular, whether in high-migrants countries welfare increases more rapidly than in relatively low-migrants countries (e.g. Barro and Sala-i-Martin 2004; Palivos and Yip 2007; Palivos 2009), and whether an empirical long-run relationship emerges between domestic welfare effect and the substitution of input productive factors (see, for example, Borjas 1994, 1995, 1999; Quibria and Islam 2010).

In this paper we reexamine the welfare effect of immigration developed by the preliminary and pioneering study of Hazari and Sgro (2003) (henceforth, HS) and afterward reviewed by Moy and Yip (2006) (henceforth, MY).

HS develop a Ramsey-type model to show that migration necessarily lowers per-capita domestic consumption if migrants and domestic residents are perfect substitutes in production.

In the MY reexamination, the welfare effect (i.e., the effect on per-capita domestic consumption) of migration is instead ambiguous as a result of the influence of two opposing effects: an intra-temporal positive effect (exploitation effect) and a negative inter-temporal effect. However, in both cases, they find that per-capita domestic consumption growth rises as a result of foreign workers.

In contrast to this result, we find a negative short-run relation between per-capita consumption growth and migrant labour. In fact, by using a Cobb–Douglas production technology properly augmented to include migrants, which corresponds to the empirically relevant case of imperfect substitution between migrants and domestic labour (e.g. Borjas et al. 2008), we find that in presence of migration, per-capita consumption growth is lower than in absence of migration; however, the long-run effect of migration on per-capita domestic consumption is positive when production is "sufficiently" reactive to capital. Therefore, in this context, economies with a higher elasticity of capital receive wider benefits from migration inflows. An economic interpretation to these results may be that changes in specialization, investment response, technological adjustment, innovation in response to migrants inflows may take some years to translate into productivity, generating some crowding out effects, more competition and costs even if this has long-run benefits. The gains for natives are likely to materialize in the long run when the specialization of natives adjusts, firms invest in capital and adopt appropriate technologies (see Quibria and Islam 2010 among others). Our analytical results are reconcilable with several empirical and theoretical studies, that the effect on per-capita consumption growth is negative in the short run and that migration flows facilitate the growth of capital-reactive economies in the long run. Furthermore, other studies analyze the effects of migration on domestic consumption by simulations. they find that migrants could generate significant welfare gains for the natives in the long run (see Ho 2007; Liu 2010).

Our contribution departs from the current literature under several respects.

First, unlike the previous studies of HS and MY, we explicitly consider the effect of migration in both the short and long run. In fact, in HS there is not an explicit analysis of long-run per-capita consumption in the context of imperfect substitution of workers as they

rather address the relationship between migration and short-run consumption growth rate (see HS, p. 150); on the contrary, in MY the only steady-state relationship between consumption and migrants is analysed in the case of perfect substitution between workers.

Our contribution generalises and complements some results of the papers cited above. In particular, we generalise the case of perfect substitution between foreign and domestic labour to the case of imperfect substitution by using an intensive form of Cobb–Douglas production function properly augmented to incorporate migrants. Furthermore, we extend the analysis to the long period, generalising and integrating in one model the results of both the previous contributions.

Finally, while previous studies examine only theoretically the impact of migrant workers on domestic consumption, the current one complements the theoretical analysis providing some empirical evidence and literature widely consistent with our results.

The remainder of paper is organized as follows.

Section 2 describes the Ramsey model augmented with migrants illustrating the theoretical structure and analyses the equilibrium properties in the short and long run. Section 3 presents some empirical evidence to support our results. Section 4 concludes.

2 The one-sector model with migrants

In this section we compare the standard version of the basic Ramsey growth model (henceforth, BRM) with Ramsey model augmented with migrant workers.

We first develop a Ramsey growth model which incorporates migrant workers; then, in order to compare it to the one-sector-Ramsey model without migration, we find the equilibrium conditions (in the short as well as in the long run) in the two models. We conclude that in the short run per-capita domestic consumption growth unambiguously decreases with migration whereas the long-run per-capita consumption of resident workers increases with migrants if the elasticity of capital is "sufficiently" high. In fact, we find a long-run positive relationship between per-capita domestic consumption and elasticity of capital.

2.1 Theoretical basic structure

We consider a one-sector growth optimizing model of Ramsey type. The economic system produces a single commodity Y using capital, native and foreign labour. The aggregate production function shows constant return to scale in physical capital K, native labour L and migrants workers M with diminishing returns of factors:

$$Y = f(K, L, M). \tag{1}$$

We note that in this formulation the production function assumes imperfect substitution between migrant and domestic labour as they enter as separate factors of production. The intensive form of the aggregate production function links output to the productive inputs in terms of total labour and can be expressed as:

$$y = f(k, l, m), \tag{2}$$

where y denotes per-capita output, i.e. $y = Y/(L + M)$, k is per-capita capital, i.e. $k = K/(L + M)$, l is percentage of domestic workers on total workforces, i.e. $l = L/(L + M)$, and m the percentage of migrant workers on total workforces $m = M/(L + M)$.

We assume that foreign workers are paid less than native workers as a reflection of labour market duality so as to capture relative disadvantages of the migrants (skill, language etc.) in specific national labour markets. Indeed, segmentation characterising labour markets in advanced industrial economies is to a large extent due to the inherent duality between domestic and foreign workers. This dualism is likely to induce systematic wage differentials for the two groups of workers (native and foreign born). In such a case we have:

$$w_m = \beta w, \qquad \text{with} \quad 0 < \beta < 1 \tag{3}$$

where w_m and w are, respectively, the migrants and the domestic wage; β captures institutional features and policies implemented in the host country as well as other factors which pin down wage differentials.

Following HS and MY we assume that migrants do not save nor accumulate capital so that the resource constraint is:

$$C = Y - \dot{K} - \beta w M, \tag{4}$$

where C is domestic consumption and the dot notation represents the time derivative.

Dividing by $L + M$ the constraint in (4) and rearranging, it yields:

$$lc = y - \frac{\dot{K}}{L + M} - \beta w m, \tag{5}$$

where $c = C/L$ is per-capita domestic consumption.

In this context per-capita profits reads:

$$\Pi = f(k, l, m) - rk - wl - \beta w m, \tag{6}$$

where r denotes the rental interest on per-capita capital k, wl and βwm are, respectively, per-capita wage bill of domestic and migrant workforces.

Profit maximization yields the following conditions:

$$f_k(k, l, m) = r, \tag{7}$$

$$f_l(k, l, m) = w, \tag{8}$$

$$f_m(k, l, m) = \beta w, \tag{9}$$

where f_k, f_l, f_m are, respectively, the marginal product of capital, domestic and migrant labour.

By virtue of the linear homogeneity of the production function, we can apply the Euler theorem and write:

$$f(k, l, m) = k f_k + l f_l + m f_m, \tag{10}$$

By using formula (10) we can compact Eqs. (7), (8) and (9) as follows:

$$w = \frac{f(k, l, m) - k f_k}{l + \beta m}. \tag{11}$$

According to HS we assume that the flow of migrants into the economy follows the rule:[1]

$$\dot{M} = (\beta w - w_0)^{\xi}, \tag{12}$$

where $0 < \xi < 1$ and w_0 is the reservation wage in the country of origin.

The growth equation for domestic workers is:

$$L = L_0 e^{nt}, \tag{13}$$

[1] This assumption is meant to draw a suitable comparison to the case of perfect substitution in HS.

which implies a positive and constant growth rate of domestic labour force $\dot{L}/L = n$, with $0 < n < 1$.

As in HS[2] when migration inflows cease in the long run, the migrant population grows at the same rate as the domestic one: $M = M_0 e^{nt}$.

By using the time derivative of $k = K/(L + M)$, after some algebra we obtain $\dot{K}/(L + M) = \dot{k} + k\,(\dot{L}+\dot{M})/(L+M)$. Substituting it into the resource constraint (5) and rearranging, we have:

$$\dot{k} = f(k, l, m) - kg(t) - lc - \beta wm, \tag{14}$$

where[3]

$$g(t) = \frac{\dot{L} + \dot{M}}{L + M} \approx n. \tag{15}$$

We are now ready to formulate the optimization problem of the representative domestic household in the Ramsey tradition (Ramsey 1928):

$$\max_c \int_0^\infty e^{-\rho t} U(c)\, dt\,, \tag{16}$$

$$\text{such that} \quad \dot{k} = f(k, l, m) - kn - lc - \beta wm\,, \tag{17}$$

$$k(0) = K_0. \tag{18}$$

The Hamiltonian associated to this problem is:

$$H = e^{-\rho t} U(c) + \lambda[f(k, l, m) - kn - lc - \beta wm], \tag{19}$$

from which, by applying the maximum principle, we derive the following first order conditions:

$$e^{-\rho t} U'(c) - \lambda l = 0\,, \tag{20}$$

$$\dot{\lambda} = -\lambda \left[f_k - n + \frac{\beta M}{L + \beta M}\, k\, f_{kk} \right], \tag{21}$$

where in the condition (21) we used equation (11) and the following relationship:

$$\frac{\beta m}{l + \beta m} = \frac{\beta M}{L + \beta M}.$$

Differentiating equation (20) with respect to time and substituting λ and $\dot{\lambda}$ into equation (21), we have the following equation:

$$\frac{\dot{c}}{c} = \frac{f_k - \rho - n + \dfrac{\beta M}{L + \beta M} k f_{kk}}{\eta(c)}\,, \tag{22}$$

where $\eta(c) = -cU''(c)/U'(c)$. Equation (22 and equation(14) are a dynamical system in c and k of Ramsey type augmented with migrant labour. In order to compare condition (22) to the case where migration does not play any role in production (BRM) in the short and the long run, we use a special case of Cobb–Douglas production function where the factor labour is a Constant Elasticity of Substitution (CES) aggregator of internal and foreign

[2] HS, pag 145.

[3] In fact, we note that $g(t) = \frac{\dot{L}/L + \dot{M}/L}{1 + M/L} = \frac{n + \dot{M}/L}{1 + M/L}$. From the reasonable assumption that $M/L \ll 1$, it follows: $1 + M/L \approx 1$ and $\dot{M}/L \approx 0$.

workers. Furthermore, we use a Constant Relative Risk Aversion (CRRA) utility function. Hence, we first compare the growth rate of consumption (22) with migrant workers to assess whether it is higher or lower than the solution without migration; then, we explore the solution in the long period (i.e., when $\dot{c}/c = 0$), to assess whether the presence of migrant workers is able to increase the long-term per-capita consumption.

2.2 Equilibrium analysis in the short and in the long run

The analytical form of the Cobb–Douglas production technology with migrants in intensive form is assumed to be:

$$f(k, l, m) = k^{\delta} h^{1-\delta}, \tag{23}$$

where $h \equiv \left[l^{\theta} + \beta m^{\theta}\right]^{\frac{1}{\theta}}$ is the CES aggregator over labour; $\delta \in (0, 1)$ is the elasticity of capital and $\beta \in (0, 1)$ is the coefficient explaining productivity differentials between native and foreign workers according to the assumed wage setting (see (3)). Moreover, $\theta \in (0, 1)$ is the elasticity of substitution between domestic and migrant workers. Also, we notice that $\theta \to 1$ involves perfect substitutability between l and m, while $\theta \to 0$ entails a low degree of substitutability between the two labour inputs. It is worth noticing that in the case of no migrant workers ($m = 0$), production collapses to the BRM case[4].

Therefore, we use a standard CRRA utility function of the type:

$$U(c) = \frac{c^{1-\alpha}}{1-\alpha}, \tag{24}$$

where $\alpha \in (0, 1)$.

By using these analytical forms explicitly, we can rewrite equation (22) in the form:

$$\frac{\dot{c}}{c} = \frac{(\delta - h_1)\gamma_1 k^{\delta-1} - \rho - n}{\alpha}, \tag{25}$$

and using equation (11) we can write the aggregate constraint (14) as:

$$\dot{k} = \gamma_1 k^{\delta} \left(1 - \frac{h_1}{\delta}\right) - kg(t) - lc, \tag{26}$$

where $h_1 := \delta(1-\delta)\beta M/(L + \beta M)$ and $\gamma_1 := \left[l^{\theta} + \beta m^{\theta}\right]^{\frac{1-\delta}{\theta}}$.

The respective case of zero migration produces the following condition on consumption growth rate [5]:

$$\left.\frac{\dot{c}}{c}\right|_{BRM} = \frac{\delta k_B^{\delta-1} - \rho - n}{\alpha}. \tag{27}$$

where $k_B = K/L$.

Now we are ready to draw a comparison between Eqs. (25) and (27). The results are summarized in the following Proposition:

Proposition 1 *The growth rate of per-capita consumption in the BRM is higher than the growth rate of per-capita consumption in the model with migrant workers.*

[4] The rationale of this formulation is basically that migrant workers are not essential to production. Therefore, the present formulation is somehow to be considered a generalization of HS Cobb–Douglas production function.

[5] The solution for the capital equation in the BRM is: $\dot{k}_B = k_B^{\delta} - nk_B - c$. See Barro and Sala-i-Martin (2004) for the development of a standard Ramsey model.

Proof we have to prove that:

$$\left.\frac{\dot{c}}{c}\right|_{BRM} > \frac{\dot{c}}{c}, \tag{28}$$

or equivalently that:

$$\frac{\delta k_B^{\delta-1} - \rho - n}{\alpha} > \frac{(\delta - h_1)\gamma_1 k^{\delta-1} - \rho - n}{\alpha}, \tag{29}$$

from which, after some simplifications, we have:

$$\delta k_B^{\delta-1} > (\delta - h_1)\gamma_1 k^{\delta-1}. \tag{30}$$

As the following relations hold by construction:

(1) $\delta > \delta - h_1 > 0$,
(2) $k_B^{\delta-1} > \gamma_1 k^{\delta-1}$ (In fact, this inequality could be equivalently rewritten as $[k(L + M)/L]^{\delta-1} > \gamma_1 k^{\delta-1}$ from which, after substituting the explicit expression for γ_1, we have that: $\frac{\delta}{\delta-h_1} > \left[1 + \beta\left(\frac{M}{L}\right)^{\theta}\right]^{\frac{1-\delta}{\theta}}$. Given that $\frac{\delta}{\delta-h_1} > 1$, the above inequality holds under the reasonable condition: $M/L << 1$ so that: $\left[1 + \beta\left(\frac{M}{L}\right)^{\theta}\right]^{\frac{1-\delta}{\theta}} \approx 1$),

we conclude that inequality (28) holds. □

It should be noticed that in Proposition 1 we are comparing growth rates along the stable arm; in fact, it is not a comparison between growth rates in steady state, but it only entails a short-run relationship between two different growth rates of consumption.

A long-run relationship is now studied comparing the steady-state consumption in the two models, as in Proposition 2 below.

In fact, imposing $\dot{c} = 0$ and $\dot{k} = 0$ in (25) and (26) we obtain the steady state value of per-capita consumption with migrants:[6]

$$c^* = \frac{1}{l}\gamma_2^{\frac{1}{1-\delta}}\left\{\left(\frac{\delta - h_2}{\rho + n}\right)^{\frac{\delta}{1-\delta}}\left(1 - \frac{h_2}{\delta}\right) - n\left(\frac{\delta - h_2}{\rho + n}\right)^{\frac{1}{1-\delta}}\right\} := \frac{1}{l}\gamma_2^{\frac{1}{1-\delta}} \cdot \Gamma, \tag{31}$$

where

$$\frac{1}{l}\gamma_2^{\frac{1}{1-\delta}} = \left[1 + \beta\left(\frac{M_0}{L_0}\right)^{\theta}\right]^{\frac{1}{\theta}} \tag{32}$$

and where Γ reads:

$$\Gamma := \left(\frac{\delta - h_2}{\rho + n}\right)^{\frac{\delta}{1-\delta}}\left(1 - \frac{h_2}{\delta}\right) - n\left(\frac{\delta - h_2}{\rho + n}\right)^{\frac{1}{1-\delta}}, \tag{33}$$

while the steady-state value of consumption obtained in the BRM is:

$$c^*_{BRM} = \left(\frac{\delta}{\rho + n}\right)^{\frac{\delta}{1-\delta}} - n\left(\frac{\delta}{\rho + n}\right)^{\frac{1}{1-\delta}}. \tag{34}$$

The comparison between c^* and c^*_{BRM} produces the following result:

[6] It is worth remembering that, in the long run, $M = M_0 e^{nt}$ and γ_1 becomes $\gamma_2 = \frac{\left(L_0^{\theta} + \beta M_0^{\theta}\right)^{\frac{1-\delta}{\theta}}}{(L_0 + M_0)^{1-\delta}}$ and h_1 becomes $h_2 = \beta\delta(1-\delta)M_0/(L_0 + \beta M_0)$. We also have that $l = \frac{L_0}{L_0 + M_0}$.

Proposition 2 *In the long run the following relationship holds:* $\lim_{\delta\to 1} c^* > \lim_{\delta\to 1} c^*_{BRM}$.

Proof When production is "sufficiently" reactive to capital changes (i.e., δ is "sufficiently" high), the steady state of per-capita consumption with migrant workers is higher than the steady state of per-capita consumption in the BRM. In other words we have to prove that:

$$\lim_{\delta\to 1}\left[\frac{1}{l}\,\gamma_2^{\frac{1}{1-\delta}}\cdot\Gamma\right] > \lim_{\delta\to 1}\left[\left(\frac{\delta}{\rho+n}\right)^{\frac{\delta}{1-\delta}} - n\left(\frac{\delta}{\rho+n}\right)^{\frac{1}{1-\delta}}\right], \tag{35}$$

where the limits are formally defined in the usual way: if we set $\lim_{\delta\to 1} c^* = v^*$ and $\lim_{\delta\to 1} c^*_{BRM} = v^*_{BRM}$, we have that for every $\epsilon > 0$ there exists $\bar{\delta}_\epsilon$ such that for every $\bar{\delta}_\epsilon < \delta < 1$ it yields

$$\left|c^* - v^*\right| < \epsilon \qquad \text{and} \qquad \left|c^*_{BRM} - v^*_{BRM}\right| < \epsilon.$$

To this aim we observe that the term Γ in (35) is a function of $\delta \in (0, 1)$ and lower than c^*_{BRM} for $\delta \in (0, 1)$. Further, the limit of Γ coincides with the one of c^*_{BRM} as $\delta \to 1$ because $\lim_{\delta\to 1} h_2 = 0$. Then we can write:

$$\lim_{\delta\to 1} c^* = \lim_{\delta\to 1}\frac{1}{l}\,\gamma_2^{\frac{1}{1-\delta}}\,\Gamma = \left[1+\beta\left(\frac{M_0}{L_0}\right)^{\theta}\right]^{\frac{1}{\theta}}\lim_{\delta\to 1} c^*_{BRM}, \tag{36}$$

from which it follows

$$\lim_{\delta\to 1} c^* > \lim_{\delta\to 1} c^*_{BRM}.$$

Hence, by virtue of the sign-preserving Theorem of limit, we can conclude that for every $\delta \in (\bar{\delta}_\epsilon, 1)$ it must be: $c^* > c^*_{BRM}$. □

This result is quite intuitive and connected with the wage setting mechanism we have assumed in (3).

In our theoretical framework firms, whose production function is characterized by a certain degree of substitutability between capital and labour (of internal and migrant type), can pay different wages to workers according to their productivity. Therefore, in equilibrium immigrants are less productive and paid lower wages than internal workers (see Eqs. (8), (9)); firms will tend to substitute internal with migrants workers recording a worsening in productivity and cheaper labour costs. The substitutability between internal and migrants workers will continue until the reduction of labour cost is greater than that of productivity: when the substitution involves a reduction of labour cost smaller than that of productivity, firms will tend to invest in capital or, in other words, to substitute labour with capital.

With a Cobb–Douglas specification the firm capital elasticity is represented by the factor δ: the impact on domestic consumption will be negative if this factor is low; the impact will be positive if δ is "sufficiently" high.

Empirical evidence on this issue is extremely rich and we widely review it in the next section.

3 Empirical evidence and related literature

The aim of this section is to provide some evidence in order to empirically support the theoretical Propositions in the previous section. It should be emphasized that our analysis

of the relationship between consumption and migrants entails an assessment of the simple correlation, rather than an attempt to establish causality links or econometric evaluations. However, the literature studying the impact of immigration on the host countries is quite large and a complete survey is beyond the scope of this paper. Hence, we will only briefly refer to a few of those studies addressing the consistency of our results with the empirical evidence.

Notably, most empirical contributions have highlighted that immigrants have adverse effects in the short run and that these effects disappear in the long period, when capital adjusts to take advantage of higher returns triggered by the inflow of migrants (among others see Peri 2010a and references therein). These studies are likely to confirm our basic intuition by showing that the impact of migrants flows on the rate of domestic consumption growth is negative in the short run and per-capita consumption is positive in the long run after some capital adjustments.

This section presents a mixture of empirical evidence employing two alternative strategies to bring the predictions of our model to empirical evidence. The first, based on a sample of international data of OECD countries, evaluates the association between average of per-capita consumption growth rates and the percentage of migrants on total labour force for a window of ten years. The second, based on single country data (i.e., US, Peri 2010a), provides empirical evidence of the long-run domestic consumption and elasticity of capital.

Table 1 sets out some information on the correlation between per-capita consumption growth rate and the stock of foreign labour force in selected OECD countries. It provides *prima facie* evidence in support to the key prediction of our theoretical model (Proposition 1).

The first column provides the average of growth rate of per-capita domestic consumption[7] over ten years (1998–2007); the second column presents the average of the stock of foreign labour force as percentage of total labour force over the same period; the third column presents the correlation index between these two variables over the same period as well.

The immediate point that stands out is the enormous differences in the European percentages of foreign labour force. These stretch from 1.9 in Finland to 20.7 in Switzerland and likely reflect country-specific migration policies and country-specific features of the labour market. A closer look at Table 1 raises two additional points. First, most European countries show a highly negative relationship between per-capita growth rate of consumption and the stock of foreign labour force (on average). This relationship is true for countries having very different migration policies and a very different structure of the labour market. Second, there are some exceptions. France, Sweden and Netherlands show a positive value of the correlation index. Their immigration policies are based on assimilation and naturalization (see Stalker 2002; Freeman 2004; Adepoju et al. 2010) and tend to facilitate naturalization of immigrants with respect to countries such as Switzerland and Austria "*characterized by labour market dominated by a guest worker migration policy*" (see Gustafsson and Zheng 2006). In Sweden and Netherlands immigrants have basically the same rights and freedoms as natives (but they cannot vote in parliamentary election) and after a few years they can apply for naturalization, even if the immigrant is unemployed; the unique prerequisite is the knowledge of, respectively, Swedish or Dutch language. France facilitates the assimilation of immigrants from ex-colonies, especially Algeria and Morocco, allowing naturalization after a few years. Immigrants from ex-colonies have usually a perfect knowledge of the French language. These migration policies produce a significant reduction in the number of migrants because of strong naturalization, and increase native workers with the consequence that the

[7] In order to smooth out both the cycle and year-on-year noise the correlation index is calculated on the permanent component of consumption by using the Hodrick–Prescott filter ($\lambda = 100$ on annual data).

Table 1 Shape correlation between growth rate of per-capita consumption and the share of foreign labour force for a sample of OECD countries

	Annual growth rate of per-capita domestic consumption (%) (1998–2007)	Stock of foreign labour force/total labor force (%) (1998–2007)	Correlation (Pearson index)
Austria	1.1	11.5	−0.99
Belgium	1.0	8.9	−0.75
Denmark	1.4	3.7	−0.30
Finland	2.6	1.9	−0.88
France	1.7	5.7	0.84
Germany	0.8	9.0	−0.44
Italy	0.8	4.8	−0.90
Greece	3.1	6.2	−0.95
Netherlands	1.3	3.7	0.30
Norway	2.9	6.0	−0.71
Portugal	1.9	3.9	−0.92
Spain	2.0	4.9	−0.98
Sweden	2.1	4.8	0.75
Switzerland	0.8	20.7	−0.60
United Kingdom	2.3	5.0	−0.97

Source our elaboration on Eurostat data

ratio $M/(L + M)$ decreases producing, in our sample, the observed positive correlation between the growth rate of consumption and the share of foreign workers.

To summarize, Table 1 provides some stylized facts in favour of a negative relation between the rate of consumption growth and the migrant labour. It also suggests that countries with a low level of migrants have a high rate of consumption growth (on average): in practice, when the share of immigrants is small, an increase in this share leads to a soft reduction in domestic per-capita consumption growth.

Another important aspect that contributes to explain the migrant flows is the business cycle. In fact, the cyclical pattern of migration is widely documented. Some estimates suggest that the inflow of immigrants in Europe as well as in US increases and decreases with the employment opportunities, and then with the business cycle (e.g. Clark et al. 2007; Hatton 2010; Hatton and Williamson 2006).

Peri (2010a) estimates the effect of immigration over the whole US business cycle suggesting that "*in the long run immigrants... increase productivity and hence average income. This finding is consistent with the broad existing literature on the impact of immigration in the United States*". On the other hand, "*the short-run impacts of immigration, however, find some mild negative effects: immigration may slightly reduce...average income...because the economic adjustment process is not immediate*" (Peri 2010a, p. 7).

These estimates for US provide useful insights to get the effects of migration in the short and in the long run (see Table 2 in Appendix). In the short run (1 year differences) migration produces some crowding out effects mainly in the recession scenario. Obviously, these effects are much reduced in expansion because of the influence of business cycle (see Peri 2010b;

Hatton 2010). On average, the GDP response[8] to net immigration rates in the short run is negative pointing at an impact near −0.38 %. In the long run (7 year difference) the impact becomes positive on both output, capital and total factor productivity (TFP) reflecting the long-run gains from immigration for natives. They are not negligible (+0,2 increase in GDP per person for an increase of immigrant population of 1 %). These results are consistent with the long-run outcome of our model (Proposition 2) and are in accord with other works (see for example Card and Shleifer 2009), confirming the positive long-run effect of immigration of the average income of Americans. Quibria and Islam (2010) show the crucial role of the innovation elasticity in determining the positive long-run impact on some macroeconomic variables in a simple growth model, confirming that in the short run the impact of immigration on economic outcomes is negative. In a similar fashion but using different techniques, many recent works confirm these findings in other countries as well. By using a market equilibrium approach Ottaviano and Peri (2006, 2008) find a positive long-run effect on some macroeconomic variables in the US with a moderate degree of substitutability between the two groups of workers; Dustmann et al. (2010) document the same thing using data for Germany. Other contributions (Ho 2007; Liu 2010) quantitatively evaluate the welfare effect of migration by using simulations. Ho (2007) calibrates the dynamic migration model by HS for eight countries finding that migration shifts up the time path of long-run per-capita domestic consumption, increasing the domestic welfare between 0.09 and 2.93 among countries supporting our theoretical results. Liu (2010) develops a dynamic general equilibrium model of labour market search type and evaluates the long-run impact of immigration on consumption. Under a benchmark calibration, the model generates a negative relationship between the population share of migrants and domestic consumption in the short run (when the population share of migrants is small) but this relation becomes positive once that the population share of migrants passes a certain threshold (i.e., in the long run).

Finally, we focus on the relation between per-capita domestic consumption and the elasticity of capital searching for a soft evidence of the relation issued from the Proposition 2. In short, our results suggest that, other things being equal, in the long run firms respond to the inflow of migrants by increasing their long-run capital-labour ratio. In practice, firms invest to change the technological structure because the flows of migrants stimulate competition, differentiation of products and efficiency inducing the adoption of appropriate technologies (see Quispe-Agnoli and Zavodny 2002). So, in the long run this creates benefits on productivity and consumption: "*Immigration's economic benefits mostly result from its effect on immigrant and native workers' occupational choices, accompanied by employers' investment and reorganization of the firm. For instance, immigrants are usually allocated to manual-intensive jobs, promoting competition and pushing natives to perform communication-intensive tasks more efficiently. This process, at the same time, reorganizes firms' structure, producing efficiency gains and pushing natives towards cognitive and communication-intensive jobs that are better paid*" (Peri 2010a, p. 6).

Accordingly, a lot of empirical contributions stress how immigration changes a country production structure. In these studies, the basic idea is that a large share of migrants may cause a reallocation of resources towards firms whose technology requires more of these workers or firm-level shifts towards less labour intensive productions and technologies (see Doms and Dunne 1998; Gandal et al. 2004). Empirical evidence corroborates quite neatly this hypothesis. Lewis (2005) and Card and Lewis (2005) show that most of the increase of the migrant population (with low productivity) from Mexico to US has been absorbed by capital changes. In practice, US firms "downgrade" their technologies when the relative

[8] Implicitly, we assume the short-run impact on income as proxy of impact on welfare.

supply of low-productive works increase (see also Dustmann et al. 2008). Gandal et al. (2004) find an analogous effect for Russian-immigration in Israel while Dustmann et al. (2010) find similar results for Germany suggesting that the technological adjustment is due to the within firm component: factor intensities, indeed, shift toward a relative more intense use of low-productivity migrant workers.

Overall, there is a broad consensus on the empirical ground both of a short-run crowding out effect on welfare consistent with our theoretical Proposition 1, and a positive effect in the long run (under some technological adjustments, consistent with our theoretical Proposition 2).

4 Conclusions

A simple model of Ramsey type with migrant labour implies that the growth rate of per-capita consumption is negatively related in the short run, while in contrast in the long run the steady-state per-capita consumption with migrant workers is higher than the steady-state per-capita consumption in the BRM, when the economy is "sufficiently" reactive to capital changes.

Our contribution generalises and extends some results of the literature on this topic (in particular, those in HS and MY). In fact, we first generalise the case of perfect substitution between foreign and domestic labour to the case of imperfect substitution by using an intensive form of Cobb–Douglas production function properly augmented to deal with immigrants.

Finally, while previous studies examine only theoretically the impact of migrant workers on domestic consumption, the current study complements the theoretical analysis providing some empirical evidence supporting our results. We employ two alternative strategies to bring the predictions of our model to data. The first, based on a sample of international data, evaluates the association between averages of rate of consumption growth and the percentage of migrants on total labour force for a window of ten years. The second strategy, based on single country data (i.e., US), provides empirical evidence of the long-run domestic consumption and elasticity of capital. Although these findings are more likely to reflect other factors as well such as institutions and migration policies, the empirical results show consistency with both the predictions of the theoretical model: lower migrants are associated to higher rate of domestic consumption growth and capital elasticity is positively correlated to steady-state domestic consumption.

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5 Appendix

See Appendix Table 2.

Table 2 Response to net immigration rates in periods of expansion and downturn in the short and long run period

	1-year differences 1994–2008		7-year differences 1994–2008	
Dependent variable	Output gap (<0)	Output gap (≥0)	Output gap (<0)	Output gap (≥0)
% Response of GDP per worker	−0.59	−0.17	0.08	0.36
	(0.18)	(0.12)	(0.39)	(0.31)
% Response of TFP	−0.57	0.01	0.24	0.47
	(0.22)	(0.17)	(0.37)	(0.45)
% Response of capital intensity	−0.85	0.01	0.51	0.43
	(0.73)	(0.38)	(0.38)	(2.01)

Source Peri (2010a,c). *Note* A technical explanation of the method of estimation and a detailed description of the methodology are contained in Peri (2010a). Heteroskedasticity- and cluster-robust standard errors are reported in parenthesis. Each regression includes time fixed effects

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