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Combining groundwater and game-theoretical models for determining the sustainable yield in a heavily stressed aquifer system

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ABSTRACT

Sustainable groundwater abstraction is nowadays widely debated and the application of criteria available in the literature is not easy depending on site-specific features of the systems. In the Tivoli Plain (Rome, Italy) the same groundwater resource is exploited by an important thermal plant for therapeutic purposes and is subject to dewatering for the extraction of travertine by numerous quarries, thus representing a significant case of conflict among different resources users. The study proposes a new approach in defining the sustainability of groundwater exploitation by matching the results of groundwater flow model with those of a game model. From the conceptual hydrogeological model and the numerical flow simulations, the sustainability conditions for the groundwater withdrawals are obtained and the hydrodynamic interferences between the different users are assessed. The results of numerical simulations were encapsulated in the game-theoretical model in order to find conceptual solutions to maximize the profit related to quarry's economy and, at the same time, to limit the quantitative and qualitative decline of the thermal waters. Assuming that the sustainability of the groundwater abstraction from the quarry area depends on ensuring residual groundwater flow to the thermal springs and the river, a cooperative game among the different players has been developed. The strategy of a player is bound by the influence of dewatering on the springs and the river and the player's utility is proportional to the total quantity of travertine extracted. Results indicate that the "price of sustainability" would be around a 23% reduction of the total exploited travertine volume, driven by an overall reduction in dewatering of about 31%, if compared to that necessary to reach the maximum depth of excavation. For the different groups of quarries the reduction of the pumping flow is not homogeneous but varies according to the position of the pumping center in the groundwater flow net and its distance from boundaries to be captured. Therefore, the combination of the flow model with the game model solutions appears to be a valuable tool to achieve sustainable groundwater management in a conflicting situation among different users. In addition, the game model allows to introduce other players and/or to find out other, non-cooperative solutions.

1. Introduction

Groundwater sustainability is nowadays widely debated (Alley and Leake, 2004; Pierce et al., 2013; Bredehoeft and Alley, 2014; Thomas et al., 2017; Gleeson et al., 2020; Elshall et al., 2020). In the literature many definitions, such as sustainable yield, sustainable groundwater development, and sustainable groundwater management are found. One of the most concise and explanatory definitions is given by Alley et al. (1999), who describe groundwater sustainability as the "development and use of groundwater in a manner that can be maintained for an indefinite time without causing unacceptable environmental, economic, or social consequences." It follows that groundwater management based on the concept of sustainability calls for a multidisciplinary approach (Pierce et al., 2013; Mays, 2013; Famiglietti et al., 2015; Thomas et al., 2017). Issues related to the amount of sustainable groundwater abstraction are certainly at the heart of such approach. In this regard, different views on the criteria underlying the definition of sustainable yield of a system are found in the literature. Many approaches are based on the comparison between the natural recharge and the pumping rate, assessing the safe yield as "the pumping rate that does not exceed the rate of natural recharge". Others consider that the pumping rate is only a function of the capture (i.e., the sum of the increase in recharge and

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decrease in discharge caused by pumping), regardless of the natural recharge. Moreover, other approaches consider the residual groundwater outflow from the system, which depends on natural recharge, and the development of the capture as criteria to define the sustainable yield (Alley et al., 1999; Sophocleous, 2000; Bredehoeft, 2002; Alley and Leake, 2004; Kalf and Woolley, 2005; Devlin and Sophocleous, 2005; Zhou, 2009; Konikow and Leake, 2014). The application of these criteria is not easy and depends on site-specific hydrogeological, environmental, economic, and social characteristics of the systems.

Mining and quarrying give considerable benefits to society, producing raw materials for industry and supporting the local economy. On the other hand, mining and quarrying can have a significant environmental impact. Groundwater is one of the environmental matrices that can be impacted by extractive activities, that represent a threat to the quality and quantity of groundwater. Extensive aquifer dewatering often necessary for mining and quarrying can cause severe impacts on local groundwater flow and discharge, and consequently on springs, streams, wetlands and their ecosystems (Raghavendra and Deka, 2015; Varouchakis et al., 2022). In such cases, in order to reduce the impact on groundwater, a dewatering management plan is required that can be designed to search the sustainable yield that maximizes the benefits of the extractive activity and limits the negative impacts on groundwater. In this regard, the plain of Tivoli (Rome) is an emblematic case in Italy.

In fact, in the Tivoli plain important and well-known thermal springs, the Acque Albule springs (Ca-HCO_3-SO_4 water with temperature of 23 °C), emerge (or rather emerged, with a flow rate up to 2 m^3/s) and supply the thermal plant Terme di Roma (annual turnover of about 8 million euros). In the same plain, another profitable economic activity concerns the extraction of travertine (revenues of about 250 million euros per year), practiced since Roman times. Since the 1970s, the appropriate excavation depth to extract travertine continuously increases, until intercepting the aquifer containing the thermal waters. The dewatering of the quarry area led to a progressive decline both in groundwater level in the travertine aquifer and in the discharge of the Acque Albule springs. As a result, the thermal plant, that was previously fed by the natural outflow of groundwater at the surface, is now fed by water pumped from the karst lake where the springs discharge. Several studies have been carried out on the hydrogeology of the Acque Albule Basin and on the impacts of dewatering (see next section). Nevertheless, the problem of the management of groundwater of the plain still remains open, bringing into conflict the two economic activities, i.e., therapeutic and mining ones.

Aim of this study is to identify a new approach to define the sustainable yield of the Tivoli Plain, where important groundwater resources in terms of quantity and quality are significantly affected by the dewatering from the quarry area. The objective is to find, under the current conditions of use of groundwater, conceptual solutions to maximize the profit related to quarry's economy and, at the same time, to limit the quantitative and qualitative decline of the thermal waters. Drawing inspiration from this specific case, the broader intent is to propose a new approach that can contribute to groundwater management in situations of conflict among water users.

To achieve this goal, the hydrogeology of the system, a hypothesis of distribution of dewatering centers and the results of a recent flow modeling implemented for the travertine aquifer were considered, in order to develop a game-theoretical model. Game-theoretical modeling (von Neumann and Morgenstern, 2007; Mazalov, 2014; Matsumoto and Szidarovszky, 2016) is a widely used approach in optimization problems for conflict situations when specific applications to water resource management are needed (e.g., Kicsiny et al., 2014a, 2014b, 2022; Kicsiny, 2017; Kicsiny and Varga, 2019).

2. Geology and hydrogeology of the study area

The Tivoli Plain, located 25 km east of Rome, covers an area of about 30 km^2 (Fig. 1a). The plain represents a subsiding, N-S fault-controlled Pleistocene basin, the Acque Albule Basin (Faccenna et al., 2008), bordered to the north and east by the Apennine belt, to the south by the River Aniene and to the west and south by the distal slopes of Colli Albani volcano (Fig. 1a). The substrate of the Acque Albule Basin consists of a thick succession of Mesozoic-Cenozoic marine carbonate rocks, faulted and folded, outcropping in the surrounding Appenine mountain ranges (Faccenna et al., 1994, 2008). The basin is filled by a discordant marine Pliocene claystone and a Plio-Pleistocene to Holocene succession of alluvial siltstones and sandstones, and lacustrine siltstones and marls, intercalated with Middle-Upper Pleistocene to Holocene pyroclastic deposits (Funiciello et al., 2003; Mancini et al., 2014; Milli et al., 2016). The top of this sedimentary succession is constituted by the Late Pleistocene Tivoli travertines (*Lapis Tiburtinus*) which form a tabular plateau with average thickness of 40–50 m; the maximum thickness of 80–90 m coincides with a main N-striking fault and the area of emergence of the Acque Albule springs. The wedge-shaped travertine units are intercalated with the pyroclastic deposits and matter-rich mudstone, and, in the southernmost area, with alluvial and fluvial deposits of the River Aniene (Billi et al., 2006; Faccenna et al., 2008, De Filippis et al., 2013; Della Porta et al., 2017; Erthal et al., 2017; Scalera et al., 2021).

The stratigraphical and structural setting of the Acque Albule Basin determines recharge, flow and discharge of groundwater in the plain, which were subject of several hydrogeological studies (Boni et al., 1986; Capelli et al., 1987; Petitta et al., 2010, Carucci et al., 2012; Brunetti et al., 2013; La Vigna et al., 2013, 2016). These studies have recognized two overlapping aquifers in the plain (Fig. 1b): *i)* a deep confined or leaky, highly transmissive aquifer (transmissivity of 10^{-2} – 10^{-1} m^2/s) hosted in fractured Mesozoic-Cenozoic carbonate rocks and *ii)* a shallow, unconfined to leaky aquifer within travertine deposits, also very transmissive (transmissivity of 10^{-2} – 10^{-1} m^2/s). An aquitard, represented by low-permeability Pliocene-Pleistocene continental, volcanic and marine deposits, divides the two aquifer layers. Flows from deep to shallow aquifer occur through the aquitard, especially in the fractured and faulted zones of the carbonate bedrock and Pliocene deposits (Fig. 1b). At regional scale and under natural conditions, a groundwater flow of about 4 m^3/s, N-S oriented from carbonate reliefs towards the plain, discharges into the springs and the River Aniene (Fig. 1a). An active groundwater circulation occurs in the travertine deposits recharged by meteoric infiltration within the plain, lateral inflow from carbonate reliefs, and rise of deep mineralized fluid from carbonate bedrock (Fig. 1b).

The main spring group of the plain is represented by the Acque Albule springs (flow rate of some cubic meters per second in the 1970s) characterized by thermal waters (temperature of about 23 °C). Water of these springs, of other minor springs (a total of a few hundred liters per second in the 1970s) and that drained from the quarries belong to the Ca-HCO_3-SO_4 type, characterized by TDS values ranging from 0.8 to 2.4 g/L and DIC from 7 to 16 mmol/kg. These values reflect the mixing of the inputs from the carbonate recharge areas, the deep Mesozoic-Cenozoic carbonate aquifer, enriched in CO_2, and other uprising fluids (Petitta et al., 2010; Carucci et al., 2012; La Vigna et al., 2013).

The natural groundwater flow setting of the plain changed over time due mainly to the travertine extraction. Actually, the continuous groundwater withdrawal necessary to mine travertine has resulted in a significant decrease in the groundwater level of the aquifer (up to some tens of meters in the quarry area from the 1970s until today) and in the discharge of the Acque Albule springs (from more than 2 m^3/s until the 1970s to almost zero since 2001; Fig. 2), as well as in a possible reduction of the groundwater flow rate discharging into the River Aniene (Del Bon et al., 2015; Brunetti et al., 2013; La Vigna et al., 2016). The impact on the travertine aquifer resulting from the quarries dewatering was analyzed in time by means of different numerical flow models addressed to compare pre- and under-development conditions (Brunetti et al., 2013; La Vigna et al., 2016; Piscopo et al., 2022). On the grounds of the results of the most recent numerical flow model (Piscopo et al., 2022) and implementing further scenarios, the constraints for the sustainable use of groundwater in the Tivoli Plain have been assessed.

3. Previous numerical groundwater flow model

The latest numerical flow model (Piscopo et al., 2022), built on the basis of the hydrogeological history of the site and on the data available in the literature and in new focused surveys, was implemented in MODFLOW 2005 (Harbaugh, 2005) and calibrated by PEST_HP (Doherty, 2015) through the graphical user interface Groundwater Vistas 8 Advanced (Rumbaugh and Rumbaugh, 2020).

The model domain covers an area of about 33 km^2 and includes three layers (Fig. 3): the travertine aquifer (Layer 1), the interlayered aquitard (Layer 2), and the deep carbonate aquifer (Layer 3). Three main scenarios were simulated representing three stress steady-state periods in sequence: the first scenario (SP1) simulates undisturbed conditions, not influenced by the massive pumping performed in the quarry area; the second scenario (SP2) refers to system conditions affected by the dewatering activities in 2008 and was used to calibrate the model together with the SP1 data; the third scenario (SP3) represents the quarry area in 2020, corresponding to its maximum extent. Specifically, the stress period SP3 represents the active quarry in 2020 simplified into 14 quarry groups, covering a total surface area of about 2.7 km^2 (Fig. 3) and

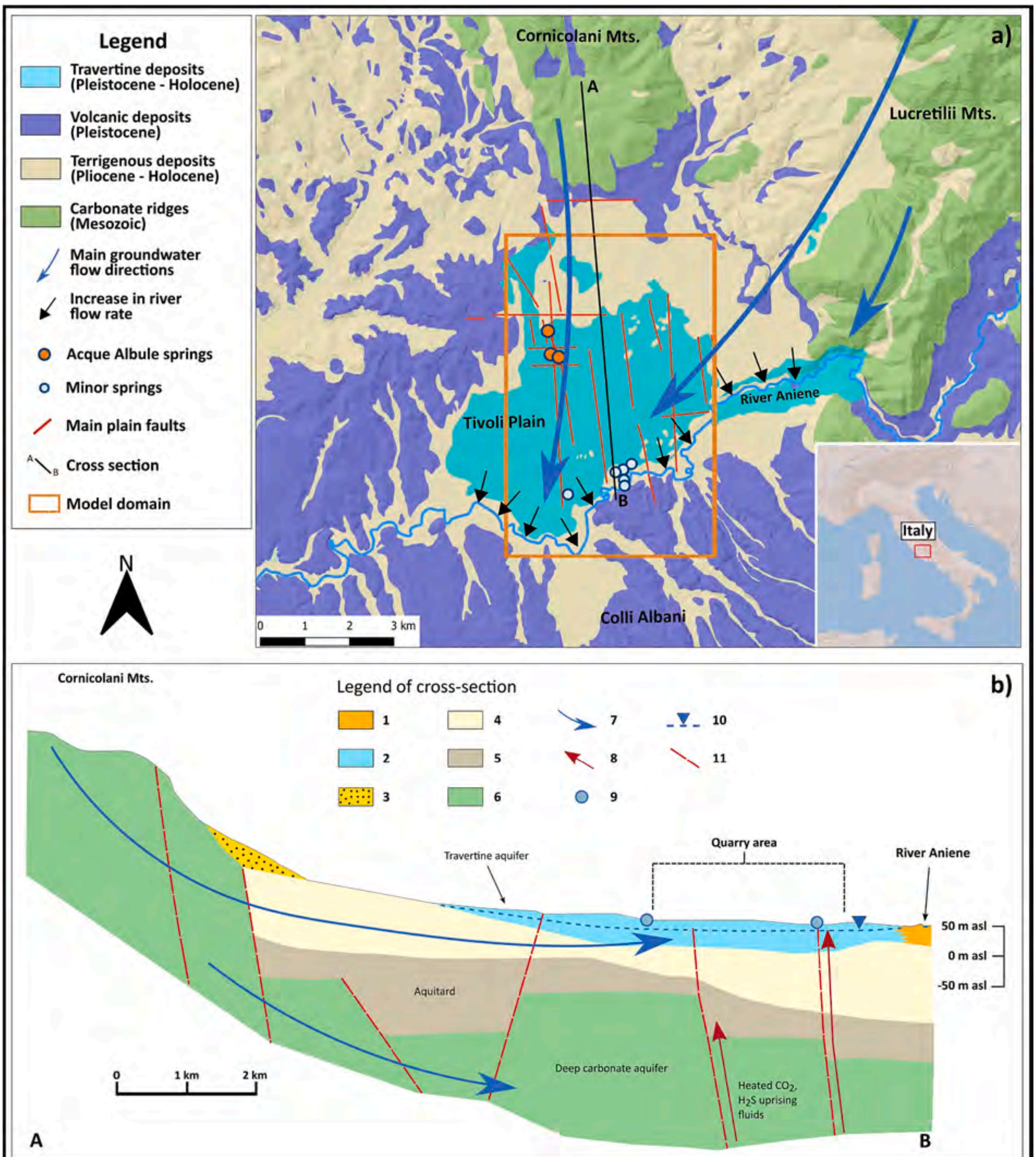

Fig. 1. Study area. a) Hydrogeological scheme of the Tivoli Plain. b) Hydrogeological cross section (A–B): 1 Alluvial deposits, 2 Travertine deposits, 3 Debris deposits, 4 Alluvial, lacustrine and volcanic deposits, 5 Claystone, 6 Carbonate rocks, 7 Groundwater flow direction, 8 Uprising fluids, 9 Springs, 10 Water table level, 11 Faults (mod. from Della Porta et al., 2017).

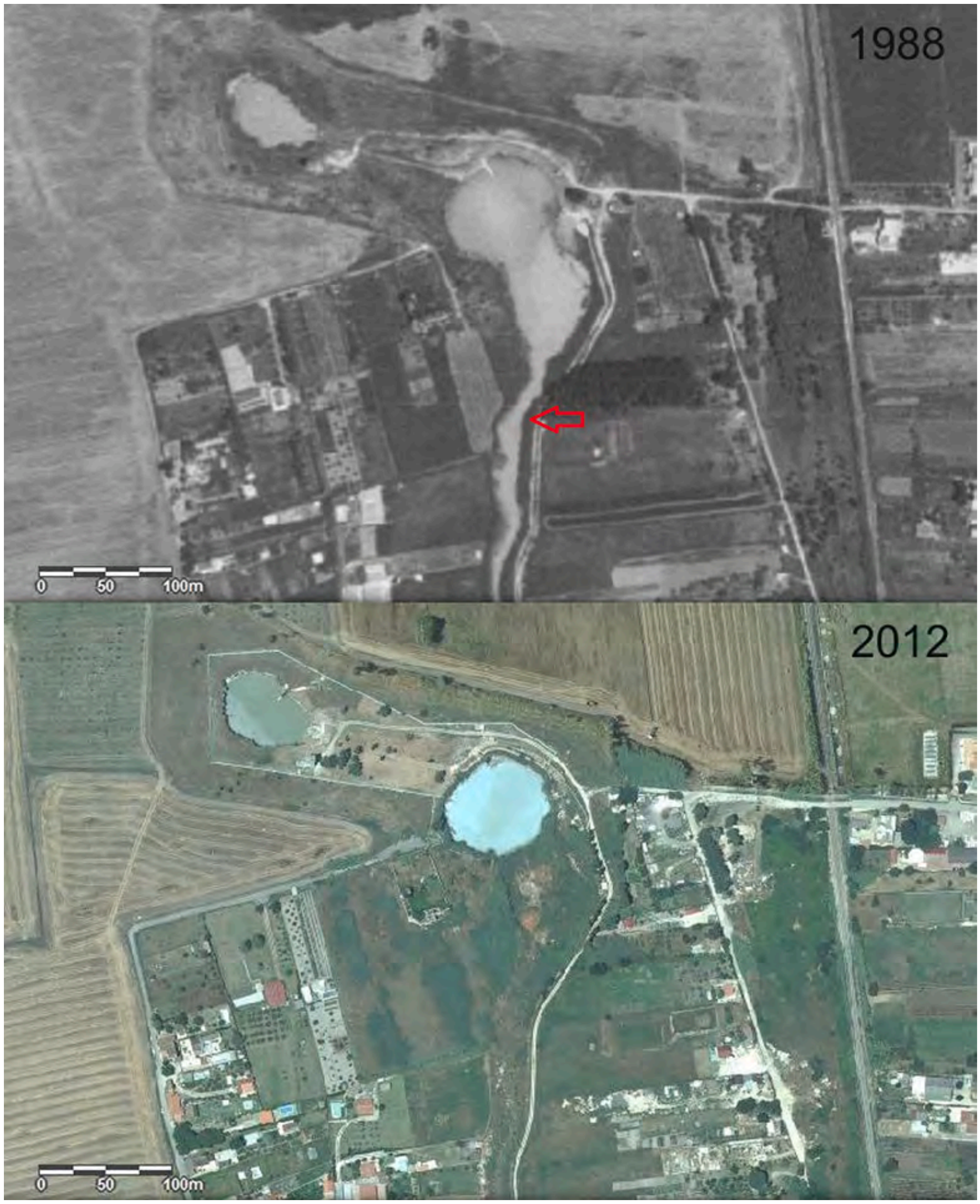

Fig. 2. Comparison of the 1988 and 2012 orthophotos of the area of the Acque Albule springs, showing the reduction in size of the discharge zone and the absence of natural groundwater outflow (highlighted by the red arrow in the 1988 picture) from the springs in 2012 (Geoportale, 2023).

characterized by a planned excavation with depths ranging between 12 and 30 m for the different groups.

The boundary conditions included (Fig. 3a): General Head Boundary (GHB), assigned to the northern limit of the model to Layers 1 and 3 and to a portion of the eastern border to simulate head-dependent inflows into the system from surrounding carbonate aquifers; No Flow Boundary, assigned to the north-western limit and to the southern sector of the model beyond the River Aniene, where null exchange between the plain and the surrounding reliefs has been recognized; Drain condition, applied to the stress periods SP1, SP2 and SP3 to the Acque Albule springs and to the minor springs and also to the quarries during the stress period SP2, the latter to simulate the dewatering; Wel package, adopted in SP3 to mimic the pumping in the quarry area; River condition, applied to the southern boundary of the model where the River Aniene flows to simulate ground-surface water interaction. The results of the coupled calibration of stress periods SP1 and SP2 allowed to determine the spatial distribution of the hydraulic conductivity of Layer 1 (horizontal hydraulic conductivity from 10^{-4} to 10^{-2} m/s) and Layer 2 (vertical hydraulic conductivity from 10^{-9} up to 10^{-5} m/s near to faulted and fractured zones) and of the conductance values of the river (10^{-3} to 10^{-1} m^2/s), drains (10^{-2} to 10^{-1} m^2/s) and GHB (10^{-3} m^2/s) boundary conditions. For Layer 3, a uniform calibrated value of hydraulic conductivity was kept (10^{-3} m/s). The obtained parameters minimize the error between simulated and observed values of hydraulic heads and flow rates and are consistent with measured transmissivity values (Piscopo et al., 2022).

The comparison between simulations under maximum dewatering (SP3) and undisturbed conditions (SP1) shows a lowering of the piezometric level in the quarry area (up to 26 m), an increase of groundwater inflow into the travertine aquifer from the northern boundary (northern GHB), a local inversion of flow between groundwater and the River Aniene, and a remarkable reduction of the flow rate of the springs (Acque Albule and other minor springs) (Table 1), as well as a reduction of the storage of the aquifer (about of 24 % of the total storage of the travertine aquifer over 40 years) (Piscopo et al., 2022). In addition, to analyze the response of the aquifer to quarry pumping rate variations,

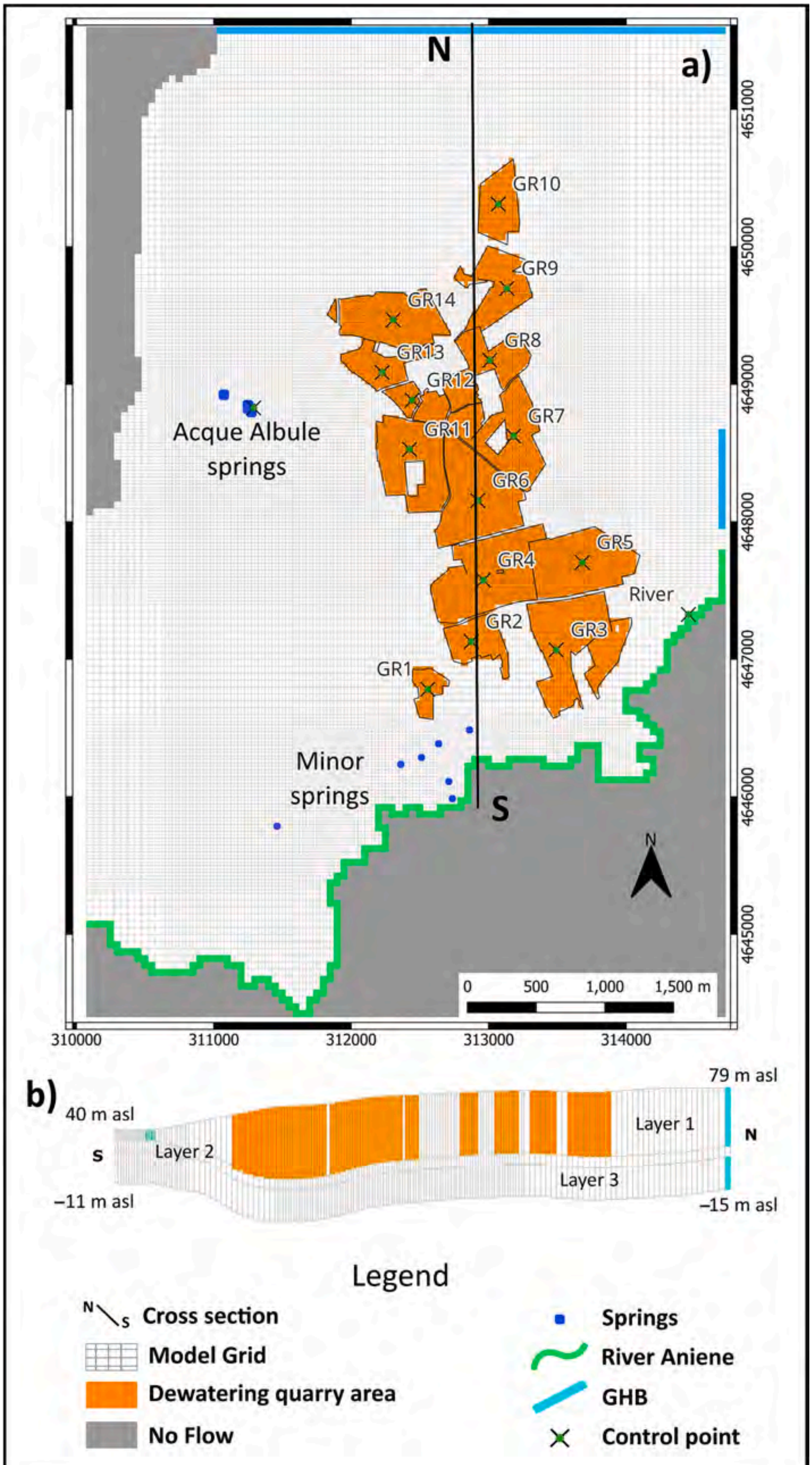

Fig. 3. Model grid and boundary conditions of groundwater flow model, plan (a) and cross section (b) views (mod. from Piscopo et al., 2022).

Table 1
Groundwater budget from the flow model (from Piscopo et al., 2022).

Term	SP1				SP3			
	Inflow		Outflow		Inflow		Outflow	
	m^3/s	%	m^3/s	%	m^3/s	%	m^3/s	%
Meteoric recharge	0.32	7.4			0.24	3.5		
Northern GHB	3.96	91.9			6.00	86.8		
Eastern GHB			0.05	1.1	0.06	0.9		
River Aniene	0.03	0.7	1.70	39.4	0.61	8.8	0.50	7.4
Quarry dewatering							5.98	88.3
Acque Albule springs			1.88	43.5			0.18	2.7
Minor springs			0.69	16.0			0.11	1.6
Total	**4.31**	**100**	**4.32**	**100**	**6.91**	**100**	**6.77**	**100**

different dewatering scenarios were simulated by varying the total pumping flow rate from the quarry area from 0 to about 6 m^3/s, that is the total flow necessary to dewater all quarry groups up to the maximum allowed excavation depth. By gradually decreasing the maximum pumping flow rate from each group of quarries, linear regressions characterized by negative slope coefficients between total pumping flow from the quarry area (ΣQ_i) and springs (Q_{AB} and Q_{M_S}) and river (Q_{River}) discharges have been found (Fig. 4), while a linear regression with a positive slope coefficient links the dewatering flow rate with the inflow from the northern GHB (Piscopo et al., 2022). On the basis of the results of this flow model, the authors identify the residual outflows into the Acque Albule springs and into the river as the main indicators to assess the sustainability of groundwater abstraction from the plain, in accordance with some lines of thought relating to the definition of sustainable yield (e.g., Kalf and Woolley, 2005; Zhou, 2009).

4. Materials and methods

To determine the sustainable dewatering rate from the quarry area a combined approach between groundwater flow and game-theoretical models has been tested in the Tivoli Plain, where conflicts among groundwater resource users occur. In the considered situation, in the game model the actors (called players) will be the quarry groups and, the strategy of each quarry group will be its pumping rate. All pumping rates together imply the drawdown in every quarry group, determining the excavation volume to be maximized. Hence the excavation volume is considered the payoff of the given quarry group. Technically, this will lead to constrained optimization problems where the sustainability conditions concerning the Acque Albule springs and the River Aniene will be considered as constraints.

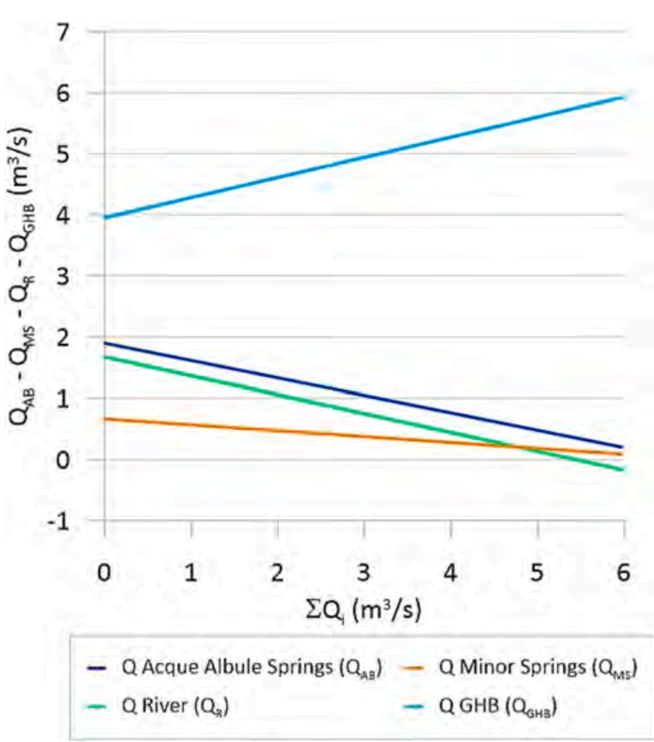

Fig. 4. Simulated variations of flow rate towards Acque Albule, minor springs, river and from northern GHB in response to variation of total flow rate from quarries (ΣQ_i).

4.1. New groundwater flow simulations

The above-described calibrated flow model (Piscopo et al., 2022) was used to carry out further simulations. These simulations were aimed at examining the influence of pumping from a single quarry group on both the sustainability indicators, i.e., discharge of the Acque Albule springs and groundwater flow from the aquifer towards the river.

The pumping rate of the single group (Q_i) was varied to observe the response of the groundwater levels, and consequently of the induced drawdowns (Fig. 5) in: *i)* the same quarry group under pumping conditions (Δh_i), *ii)* the other 13 groups that are not pumping (Δh_n), *iii)* a control point near the thermal springs (Δh_{AB}), and *iv)* a control point near the river (Δh_R) in the stretch most sensitive to the inflow from the river towards the aquifer under the SP3 scenario (Fig. 3). In detail, the simulations run varying the pumping flow rate of the single group with 0.5, 0.7, and 1 as multipliers of Q_{imax}, that correspond to the flow rate under the SP3 scenario, simulating the simultaneous pumping from all groups necessary to reach the depth of the planned excavation in all the groups. Q_{imax} depends on the depth and surface area of the group of quarries, the local transmissivity of the aquifer, and the distance of the group from the boundaries to be captured from dewatering (springs, river, and northern GHB).

The values of Q_{imax} considered in the game-theoretical model are shown in Table 2, together with the surface area of the groups and the drawdown necessary to reach the planned excavation depth. The new simulations were aimed at identifying, in addition to the mutual influences between the pumps from the 14 quarry areas, the Maximum Sustainable Drawdown in control point near to the Acque Albule springs ensuring a sufficient flow from the springs ($M\Delta h_{AB}$) and the Maximum Sustainable Drawdown in the control point near to the river ensuring residual groundwater flow to the river ($M\Delta h_R$).

4.2. Game-theoretical model

4.2.1. Model building

The drawdown of the groundwater level in a given quarry group is the result of pumping in all quarry groups. Furthermore, the volume of the extracted travertine is considered approximately proportional to the flow rate dewatered from the given excavation area. Hence every quarry group is interested in the maximization of the flow of water pumped in this quarry group. Therefore, there is a conflict situation among different

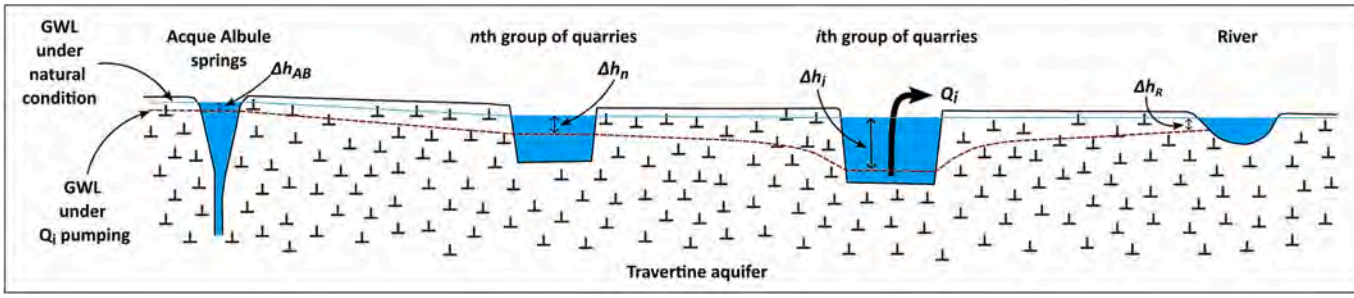

Fig. 5. Parameters observed during new groundwater simulations (cross-section not in scale).

Table 2
Main parameters of the groups of quarries.

Group	Surface area A_i ($\times 10^4$ m^2)	Δ (m)	Q_{imax} (m^3/s)
1	4.89	18.2	0.47
2	11.22	23.3	0.66
3	33.05	24.5	0.89
4	31.75	23.6	0.38
5	27.19	26.8	0.63
6	28.80	19.9	0.20
7	25.64	20.2	0.23
8	16.94	19.6	0.32
9	12.48	21.4	0.50
10	11.83	21.4	1.12
11	25.10	14.1	0.18
12	7.91	14.7	0.18
13	11.07	13.2	0.08
14	26.87	13.6	0.08
Total	274.72		5.92

Δ thickness of water to be dewatered to reach the planned excavation depth.

quarry groups, where a game-theoretical model can be applied. In addition to the standard normal form game, constraints describing sustainability conditions related to the Acque Albule springs ($M\Delta h_{AB}$) and the River Aniene ($M\Delta h_R$) will be considered.

At this point we introduce the usual terms and notation of game theory: the quarry groups are considered *players* (numbered $i = 1, \cdots, 14$). Every player i chooses a pumping rate $x_i = Q_i$ as *strategy*. Vector $x = (x_1, \cdots, x_{14})$, called *multi-strategy*, determines the drawdown $\Delta h_i(x_1, \cdots, x_{14})$ in quarry group i. If A_i is the excavation area of the latter, then:

$$f_i(x_1, \cdots, x_{14}) = \Delta h_i(x_1, \cdots, x_{14}) A_i \quad (1)$$

is considered the gain or *payoff function* of player i. Here, for the multi-strategy $(x_1, \cdots, x_{14})$, we have:

$$0 \leq x_i \leq Q_{imax} \quad (2)$$

where Q_{imax} is the maximum (possible) pumping rate in the ith group.

Sustainability conditions described by the drawdown at the Acque Albule springs and at the River Aniene are also dependent on the pumping rates x_i, and we have the following two sustainability constraints for them:

$$\Delta h_{AB}(x_1, \cdots, x_{14}) = g_1(x_1, \cdots, x_{14}) \leq M\Delta h_{AB} \quad (3)$$

$$\Delta h_R(x_1, \cdots, x_{14}) = g_2(x_1, \cdots, x_{14}) \leq M\Delta h_R. \quad (4)$$

Now, the strategy choice of the players is limited by these constraints. Therefore, we define the *set of admissible multi-strategies*:

$$G = \{(x_1, \cdots, x_{14}) \in [0, Q_{1max}] \times \cdots \times [0, Q_{14max}] | g_1(x_1, \cdots, x_{14}) \leq M\Delta h_{AB}, g_2(x_1, \cdots, x_{14}) \leq M\Delta h_R \}$$

4.2.2. Cooperative solutions of a game

The idea of a simultaneous maximization of all payoff functions leads to a constrained *vector (or multi-criterial) optimization problem*:

$$F(x) \rightarrow \max(x \in G),$$

$$F = (f_1, f_2, \cdots, f_{14})$$

A *cooperative solution* of the game is based on the concept of *Pareto optimality*. Keeping the above notation also for the general case of n players, let G be a subset of $\mathbf{R}^n$ and $F = (f_1, f_2, \cdots, f_n)$ a function mapping G into $\mathbf{R}^n$.

Point $x^* \in G$ is called *Pareto optimal* for F (and the corresponding function value $F(x^*)$ is *called Pareto optimal value*); if there is no other multi-strategy:

$$x \in G \text{ such that } f_i(x^*) \leq f_i(x) \ (i = 1, 2, \cdots, n)$$

and at least for one i the inequality is strict. If G is interpreted as the set of admissible multi-strategies, $(f_1, f_2, \cdots, f_n)$ are the payoff functions, and x^* is also called a *cooperative solution of the n-person game.* Cooperative solution means that there is no other strategy choice that makes one player better off without making some other player worse off.

The set P of the above Pareto optimal function values is called the *Pareto frontier of function F* (Blasco et al., 2008):

$$P = \{F(x) | x \in G \text{ is Pareto optimal for } F\}$$

4.2.3. Scalarization

Let S^n be the interior of the standard simplex in $\mathbf{R}^n$, that is, the set of all vectors $(\lambda_1, \cdots, \lambda_n)$ with positive coordinates summing to 1. Then it is straightforward to check that for all $\lambda \in S^n$, any solution of the scalar optimization problem:

$$\lambda_1 f_1(x) + \cdots + \lambda_n f_n(x) \rightarrow \max (x \in G). \quad (5)$$

is Pareto optimal for F. In practice, except for some degenerate cases, most Pareto points can be obtained by the above scalarization (see Geoffrion, 1968). Using the Pareto points, the Pareto frontier of F can be clearly generated.

From the above construction it is plain that, in general, an infinite number of cooperative solutions can be obtained by scalarization. In fact, as the illustrative case of Fig. 6b shows, that the Pareto frontier is the "north-eastern border" of the range of function f.

Considering a game, there should be a reasonable way to single out a unique cooperative solution. Hereafter we adapt the construction of the so-called nearly ideal solution (from Kicsiny et al., 2022).

4.2.4. Nearly ideal solution of the game

The following construction was proposed by Salukvadze (1971a, 1971b) for dynamic vector optimization problems, and soon was easily adapted to the static case. First, for every fixed $i = 1, 2, \cdots, n$, we solve the scalar optimization problem:

$$f_i(u) \rightarrow \max (u \in G) \quad (6)$$

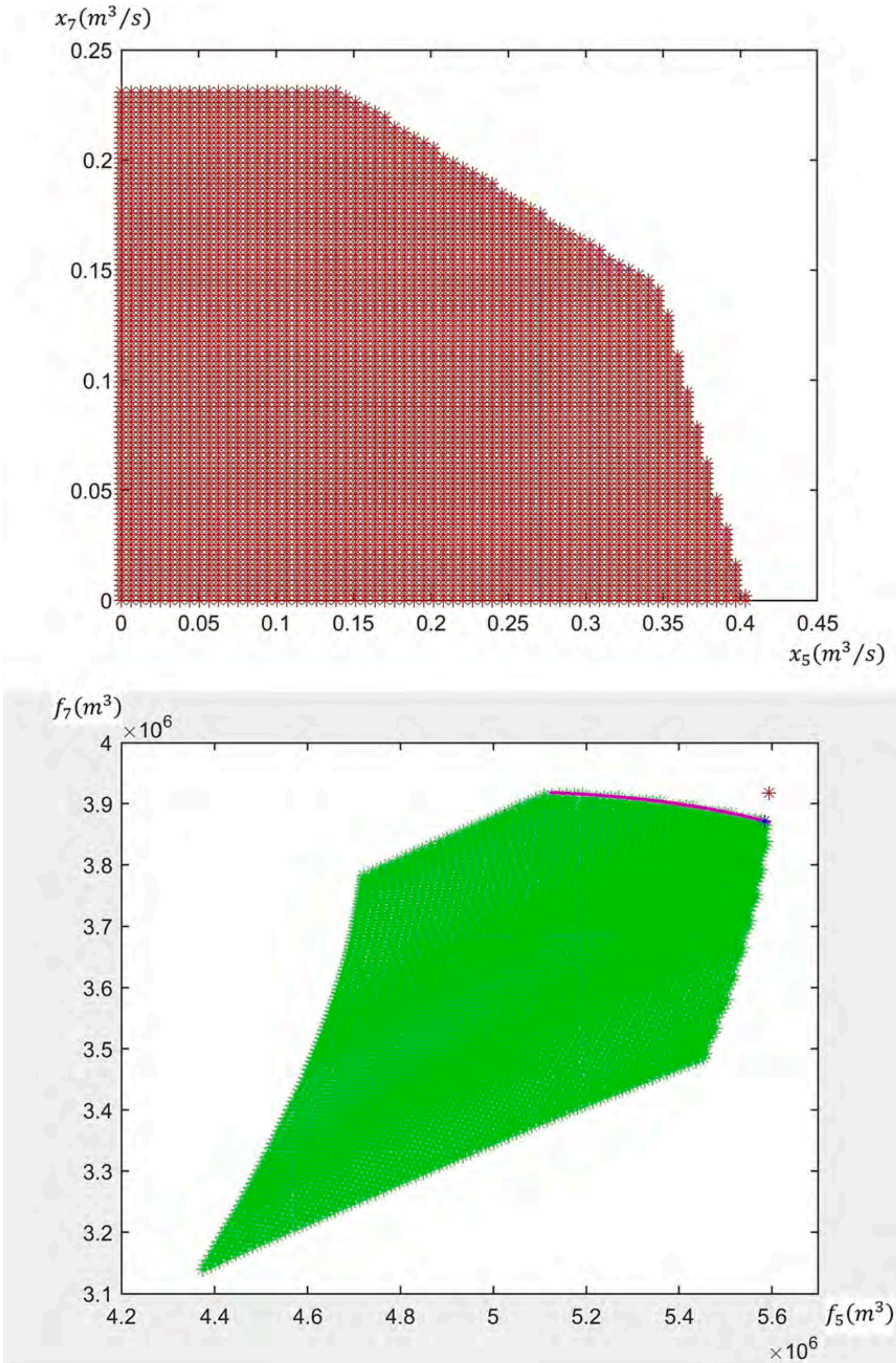

Fig. 6. **a)** The red convex polygon displays the set of strategy pairs admissible by the sustainability constraints, playing only players 5 and 7. **b)** The green set is image of the red polygon: it consists of all payoff vectors corresponding to the admissible strategy pairs of Fig. 6a. The magenta line is the set of Pareto optimal values (Pareto frontier), the red and blue asterisks are the ideal value and the nearly ideal value of the game, respectively.

where $(i = 1, \cdots, n)$, that is, we maximize the payoff of player i considering all admissible multi-strategies. Suppose that $u_i^* \in G$ is a solution of problem (6) and define $\omega_i = f_i(u_i^*)$. Then vector:

$$\Omega = (\omega_1, \cdots, \omega_n) \tag{7}$$

is called the *ideal value of the game*. Of course, in general, there is no multi-strategy where the ideal value would be attained. Nevertheless, in the Pareto frontier we can seek a point nearest to the ideal value. If for some multi-strategy $u^0 \in G$, we have $F(u^0) \in P$, and $F(u^0)$ is a solution of the optimization problem:

$$d(p, \Omega) \rightarrow \min(p \in P) \tag{8}$$

then u^0 is called a *nearly ideal solution of the game* (here d denotes the Euclidean distance). $F(u^0)$ can be called the *nearly ideal value of the game*.

It is to be noted that in the application presented below, the mathematical properties of the functions involved in the model will imply that optimization problems (5) and (6) have solutions. In the numerical realization of the above game-theoretical model (after discretization), optimization problem (8) will also have a solution.

4.2.5. Data, model fitting and numerical realization

For the definition of the constraints and the payoff functions, the data of Q_{imax} and A_i reported in Table 2 were used.

In order to solve the (scalarization of the) vector optimization problem, we need mathematical formulae for the payoff functions f_i, as well as for the constraint functions g_1 and g_2, for which it is sufficient to have formulae for the drawdown for each quarry group as well as that of the spring and of the river:

$$\Delta h_i (i = 1, \cdots, 14), \text{ or } i = AB, \text{ or } i = R$$

as functions of the multi-strategy $(x_1, \cdots, x_{14})$ (see formulae (2)–(4)).

From the new numerical simulations of the groundwater flow the above drawdown values were computed with certain preset multi-strategies (pumping rates). For example, in the SP3 scenario all groups pump at maximum rate (maximal drawdown), in SP1 scenario there is zero pumping (with zero drawdown). The effect of pumping in each single group was assessed, too. In these cases, the pumping rate in each of the quarry groups was varied from 0 to its maximum (Q_{imax} in Table 2), while there was no pumping in the other groups.

5. Results

5.1. Groundwater flow model results

A total of 43 simulations allowed to define the relationships between Q_i of the single group (in the $0 - Q_{imax}$ range) and the drawdown induced in the Acque Albule springs and in the river, the two main indicators of groundwater sustainability. The results of the simulations listed in Appendix A were used in the mathematical model assuming the following sustainable constraints in terms of drawdown when all the quarries are active:

- Maximum Sustainable Drawdown in control point near to the thermal springs $M\Delta h_{AB} = 1.27$ m, corresponding to the Minimum Sustainable Level ($MSL_{AB} = 61.44$ m asl) in the thermal springs ensuring a flow from the springs of about 0.7 m^3/s; this is the flow rate necessary to feed the thermal plant with all quarries active and then considering the superposition principle;
- Maximum Sustainable Drawdown in the control point near to the river $M\Delta h_R = 3.84$ m, corresponding to the Minimum Sustainable Level ($MSL_R = 43.43$ m asl) ensuring residual groundwater flow to the river, where it is most sensitive to the reversal of the flow under pumping from all groups.

These drawdown constraints correspond to flow rate values that can guarantee the correct operation of the thermal plant and a flow from the aquifer towards the river, clearly under the necessary simplifications adopted in the translation of a complex conceptual model into a groundwater flow model. This information was encapsulated in the game-theoretical model, assuming that the maximum profit of the miners depends on the pumping flow, that determines the volume of extractable travertine, while the maximum dewatering flow depends on the aforementioned constraints of $M\Delta h_{AB}$ and $M\Delta h_R$.

5.2. Game theoretical model

5.2.1. General results

In order to produce mathematical formulae for the drawdowns as functions of the multi-strategy, quadratic regression was applied to the data set of the drawdowns in Appendix A. More precisely, we used the following regression function for each index i, where:

$$1 \le i \le 14, \ i = AB, \text{ or } i = R. \tag{9}$$

Since we have 44 observations and 29 free coefficients, this kind of regression problem is mathematically correct. Results of the regression can be found in Appendix B. The corresponding correlation coefficients (R values) are mostly very close to 1, showing a fairly good approximation.

The numerical realization of the Pareto frontier results in a finite "approximation" obtained from a finite number of scalarized optimization problems. In fact, we have to choose a reasonable number of discrete points, which are "quasi-uniformly distributed" in $\boldsymbol{S^n}$. If $r > n$ is a positive integer, then it is obvious to pick all possible n-tuples, for which each coordinate is an integer multiple of $1/r$. The number of such n-tuples is $\binom{r-1}{n-1}$.

The numerical solution of the above constrained scalar optimization problems was carried out with the Matlab software (Etter et al., 2004), particularly using its *fmincon* function. The results are shown in Table 3. In the first column we used verbal description of the quantities at issue. Firstly, from (7) we calculated the ideal value of the game Ω.

Then we generated Pareto optimal solutions in case $r = 20$, that is, for $\binom{19}{13} = 27,132$ different λ vectors. From these points we have chosen the *nearly ideal value* of the game, $F(u^0)$, and the corresponding multi-strategy. Considering uniform weights $\lambda = (\frac{1}{14}, \cdots, \frac{1}{14})$, we determined the maximal total payoff and the corresponding multi-strategy. In Table 3, Q_max is the same as the last column of Table 2. The 8th row contains the weights of scalarization corresponding to the nearly ideal value. In the last 2 rows there are the payoff values (and the total payoff) of the SP3 scenario belonging to the (non-admissible) multi-strategy, when all groups pump at maximum rate. Values are calculated with data of simulations and regression, respectively.

5.2.2. Illustrative case of 2 players

Although there is no appropriate geometric illustration for the case of 14 quarry groups (14 players), the fictitious case of two players can be illustrated quite well. In this case only 2 selected groups take part in the game. The others do not, that is, their pumping rates are set to be constant. Now the set of all admissible strategies and the corresponding payoff vectors can be illustrated on 2 dimensional plots.

First, formally let us fix $x_i = 0.7Q_{imax}$ for $i \ne 5, 7$ (the pumping rates are considered constant for the other quarry groups) and consider that only players 5 and 7 play. Only pairs (x_5, x_7) are considered multi-strategies. Fig. 6a shows the set of all admissible multi-strategies (red region).

Without the sustainability conditions this would be a rectangle defined by the respective maximum pumping rates (for players 5 and 7, these equal to 0.63 and 0.23 m^3/s, respectively, see Table 2). In Fig. 6b

Table 3
Results of the game-theoretical model.

Quarry group	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Sum
ideal value (m^3)	76.37×10^4	2.19×10^6	6.60×10^6	6.17×10^6	5.87×10^6	4.68×10^6	4.19×10^6	2.72×10^6	2.34×10^6	2.30×10^6	2.80×10^6	91.78×10^4	1.17×10^6	2.99×10^6	45.71×10^6
nearly ideal value (m^3)	67.63×10^4	2.07×10^6	6.36×10^6	5.94×10^6	5.67×10^6	4.48×10^6	3.98×10^6	2.47×10^6	1.97×10^6	1.83×10^6	2.59×10^6	84.33×10^4	1.06×10^6	2.66×10^6	42.61×10^6
maximal total payoff (m^3)	70.71×10^4	2.14×10^6	6.41×10^6	6.10×10^6	5.70×10^6	4.61×10^6	4.08×10^6	2.49×10^6	1.89×10^6	1.53×10^6	2.69×10^6	85.40×10^4	1.06×10^6	2.60×10^6	42.88×10^6
nearly ideal solution (m^3/s)	0.28	0.60	0.46	0.35	0.40	0.19	0.16	0.23	0.33	0.78	0.10	0.08	0.05	0.05	
multi-strategy corresp. to the maximal total payoff (m^3)	0.32	0.65	0.41	0.38	0.38	0.20	0.20	0.26	0.38	0.50	0.14	0.11	0.06	0.06	
Q_max (m^3/s)	0.47	0.66	0.89	0.38	0.63	0.20	0.23	0.32	0.50	1.12	0.18	0.18	0.08	0.08	
weights of scalarization corresp. to the nearly id. value	0.05	0.05	0.05	0.10	0.05	0.05	0.05	0.10	0.10	0.15	0.05	0.05	0.05	0.10	
abs. maximum payoff (m^3, based on simulation)	89.04×10^4	2.61×10^6	8.11×10^6	7.49×10^6	7.29×10^6	5.73×10^6	5.18×10^6	3.32×10^6	2.67×10^6	2.53×10^6	3.54×10^6	1.16×10^6	1.46×10^6	3.65×10^6	55.65×10^6
abs. maximum payoff (m^3, based on regression)	88.95×10^4	2.61×10^6	8.10×10^6	7.48×10^6	7.27×10^6	5.73×10^6	5.17×10^6	3.32×10^6	2.67×10^6	2.53×10^6	3.54×10^6	1.16×10^6	1.46×10^6	3.65×10^6	55.58×10^6

the green region represents the set of all payoff vectors belonging to the admissible strategies, in other words, the image of the red region of Fig. 6a with respect to the vector function *F* (it also reflects the nonlinearity of the payoff functions).

The magenta line in Fig. 6b shows the Pareto frontier, which typically is the north-eastern segment of the boundary of the green region. On this line the nearly ideal value of the game is indicated by a blue asterisk. This point has the minimum Euclidean distance from the ideal value, which is situated outside the green region (indicated by red asterisk).

Actually, the vector connecting the red and blue asterisks would be orthogonal to the Pareto frontier if the aspect ratio of the coordinate system was equal to 1.

6. Discussion

The study carried out in the Tivoli Plain allowed to test a new approach for the possible definition of the sustainable groundwater resource management in an area where strong conflicts in withdrawals from the same aquifer occur. The results of the flow model interfaced with those of the game-theoretical model seem to be useful in finding a reasonable balance between the maximum gain for the activities of travertine extraction and the sustainability of groundwater abstraction. It should be noted that the results of this study do not represent an operational solution to the problem of the hydrogeological imbalance of the plain. Rather, they represent an approach that can be used as hydrogeological basis to guide decisions in the sustainable management in areas where a conflict between different users exists. In fact, the study considers only the current way of using groundwater, when others could also be envisaged, and is based on a reconstruction of the hydrogeological model and on quarry projects derived from data not always complete. The specific application of this approach required simplification of the conceptual model and selection of the stakeholders involved in the management of groundwater resources. Nothing prevents us from considering other players' interests (such as water management agencies, local communities and citizens' associations) and other resource management strategies (linked, for example, to a new plan of the quarry's workings or the aquifer recharge with water pumped from the quarries). In any case, also all other impacts arising from groundwater development, such as for example the influence on groundwater dependent ecosystems (e.g., Goldscheider, 2019), are primarily a function of drawdown response to pumping, which in turn is related to pumping time and to the time constant of the system (Theis, 1940). In other words, the subject matter for discussion is the proposed approach rather than the specific results related to the case study. Therefore, results obtained from the modelling-methodological development can represent an example of the underlying philosophy. The proposed approach takes into account the following logical sequence: *i)* from the conceptual hydrogeological model and the numerical simulations of the flow, the sustainability conditions for the groundwater withdrawals are obtained, *ii)* from the numerical flow model the hydrodynamic interferences among the different users are assessed, and *iii)* the possible solutions in the mathematical game are identified, respecting the economic and hydrogeological sustainability of the abstractions from the aquifer.

Therefore, considering the aforesaid assumptions, the results of the game model contained in Table 3 can be discussed. First, we can check our model whether the obtained numerical results conform to some intuitive considerations. It is clear that each coordinate of the ideal value must be greater than or equal to the corresponding coordinate of the nearly ideal value, which apparently holds in the Table 3. Also it is to be noticed that the maximal total payoff 42.88×10^6 m^3 is a little greater than 42.61×10^6 m^3, the sum of the payoffs of the nearly ideal value.

If the players intend to further cooperate, the maximization of the total payoff also may be an issue. Then the total payoff can be redistributed according to an agreement. In Table 3, as expected, the sum of coordinates of the maximal total payoff is greater than that of the nearly ideal value, and by coordinates, the ideal value is greater than or equal to the maximal total payoff. For a comparison, in the last 2 rows there are the payoff values (and the total payoffs) of the S3 scenario belonging to the (non-admissible) multi-strategy, where all groups pump at maximum rate. These values were computed in two different ways: on the basis of the results of the hydrogeological simulation (9^{th} row) and by means of regression (10^{th} row). Therefore, the total payoff, considering for example the simulated value 55.65×10^6 m^3, is substantially greater than that we can achieve under the sustainability constraints (42.88×10^6 m^3). Hence the "price of sustainability" would be around a 23 % reduction of the total exploited travertine volume. Applying our model, the price of sustainability can be estimated for stricter and less strict sustainability constraints.

The example of a solution deriving from the nearly ideal condition provides for an overall reduction in dewatering of about 31 % from the quarry area, equivalent to a total pumping flow of approximately 4 m^3/s. It is worth to note that for the different groups of quarries the

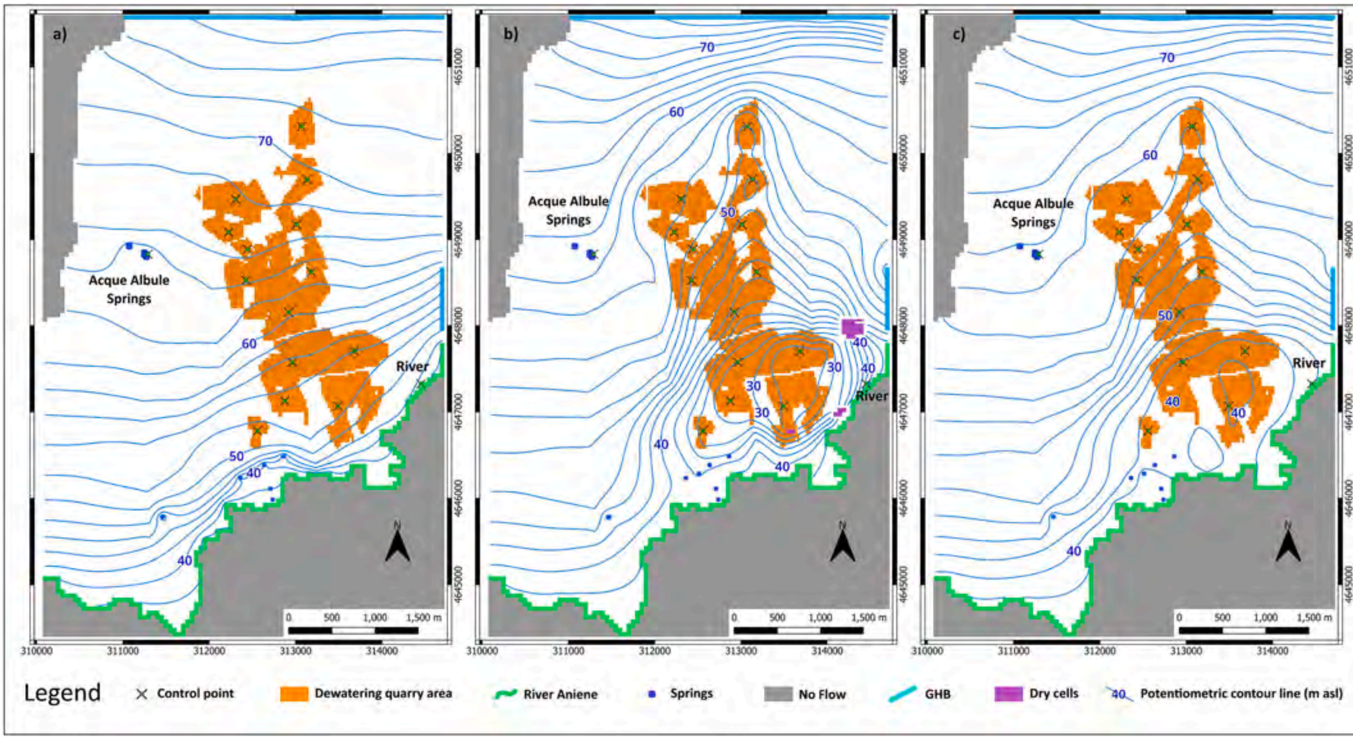

Fig. 7. Comparison of the potentiometric surfaces of the travertine aquifer: **a)** without dewatering from the quarries (SP1 scenario), **b)** under maximum pumping from the quarries (SP3 scenario) and, **c)** under sustainability conditions (nearly ideal scenario).

reduction of the pumping flow is not homogeneous, as in the case of the reduction of the volume of travertine to be extracted to maintain the sustainable residual outflow towards the springs and the river, but ranges from a minimum of 4 % to a maximum of 54 %. This finding is consistent with the results of the groundwater flow model, highlighting that the position of the pumping center in the groundwater flow net and its distance from boundaries to be captured becomes crucial (Piscopo et al., 2022). In order to represent the example of solution found under nearly ideal condition, a scenario applying the proposed pumping flow rate to each group of quarries was simulated through the numerical flow model, obtaining a new potentiometric surface. In Fig. 7 the three potentiometric surfaces obtained from the three most significant scenarios are compared: a) natural conditions (no dewatering from the quarries, SP1 scenario), b) not sustainable conditions (maximum pumping from the quarries, SP3 scenario) and, c) respectful of the conditions of sustainability (nearly ideal scenario).

From the comparison of the three potentiometric surfaces, although in the nearly ideal scenario the conditions ensuring the residual outflow to the springs and the river are generally respected, it is still evident how the quarries in the southern sector closer to the river affect the surface–groundwater equilibrium.

In the present paper we dealt with cooperative solutions of our game-theoretical model. It might be interesting to find out which are the non-cooperative solutions. It is to be noted that inKicsiny (2022), thanks to the particular mathematical form of the game constrained by specific sustainability conditions, the obtained cooperative solution also turned out to be non-cooperative solution (Nash equilibrium). We recall that a multi-strategy $(a_1,\ldots,a_n)$ is called a non-cooperative solution of the game, if no player i can obtain a higher payoff by choosing a strategy different from a_i, provided that every other player j sticks to a_j. In a future extension of our game model, if environmental and management authorities are also considered as players, then it would be interesting to see how the price of sustainability will change in the case of non-cooperative solution.

In the game model of sustainable quarrying activity we used a so-called "one-shot game" where the players choose strategy only once. Now, considering repeated games, it would be interesting to see what happens if in our case the quarry groups would also change their strategies from game to game. Now if we describe a step-by step approximation of inequalities of required sustainability constraints, we will approach the required values in the right-hand sides of inequalities (3) and (4). Then the considered game can be played several times, and the involved companies may adapt themselves to the stricter sustainability requirements.

Concerning the modelling methodological aspect of our study, further versions and generalizations of the nearly ideal (or Salukvadze) solution of a game could be also applied (see Yu and Leitmann, 1974; Li, 1990). For a somewhat different approach to multi-criterial optimization see Varga (1978). Finally, to carry on the present research, a model should be developed for the case of different locations of pumping centres in the groundwater flow net (also considering other different plans for the travertine extraction) which simulates the travertine extraction in transient conditions (i.e., considering the different phases of the stone extraction from the quarries to reach the maximum depth of excavation).

7. Conclusions

The study has allowed an assessment of possible groundwater sustainable yield the Tivoli Plain, where there are important economic activities supported by the use of groundwater and conflicts among the groundwater users themselves. The logical path firstly included the definition of indicators of hydrogeological sustainability, derived from the conceptual hydrogeological model and the flow model. These indicators have been identified in the residual outflow towards the Acque Albule springs and the River Aniene, that are the main areas of natural discharge of the travertine aquifer strongly affected by the quarry dewatering. Subsequently, following the flow model results, the residual outflow thresholds were assessed. Finally, the results of the flow model were used for the cooperative game model in order to obtain the possible maximum extraction depths dependent on the amount of dewatering. The study has therefore allowed to identify the constraints of the quarry activity taking into account the hydrogeological balance of the plain.

The tested approach, although based on a simplified definition of the concept of sustainable yield which is assumed dependent only on the local hydrogeological balances and on the economic aspects deriving from the quarrying activities, seems to be promising. Matching the flow model with the game model solutions represents indeed the novelty of the study. This combination makes it possible to estimate “the cost of sustainability” in terms of reduced income for the quarry groups. The game model allows us to introduce other players, in addition to miners, such as citizenship, public bodies responsible for the groundwater management, users of thermal waters for therapeutic purposes and so on, becoming a valuable tool to achieve participatory and sustainable resource management. Moreover, other plans of extraction of the travertine and management of the groundwater resources can be tested through the used approach; however, this would require additional data currently not available for the analysed system.

The modelling approach used associated with a suitable monitoring system, will give operational answers also in other situations of conflict among different groundwater users, being the integration between coupled flow-game models and monitoring the key to understand how the aquifer responds to human impact, followed by the search for sustainable operational solutions.

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CRediT authorship contribution statement

Vincenzo Piscopo: Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Funding acquisition, Formal analysis, Conceptualization. **Chiara Sbarbati:** Writing – review & editing, Writing – original draft, Validation, Formal analysis, Data curation, Conceptualization. **Zoltán Sebestyén:** Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Funding acquisition, Formal analysis, Conceptualization. **Zoltán Varga:** Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Funding acquisition, Formal analysis, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A:. Results of new groundwater flow simulations

Drawdowns (in m) to 16 control points (Δh_i, Δh_{AB}, Δh_R) during pumping from the single group with flow rates of 0.5, 0.7 and 1 Q_{imax}.

		Δh_1	Δh_2	Δh_3	Δh_4	Δh_5	Δh_6	Δh_7	Δh_8	Δh_9	Δh_{10}	Δh_{11}	Δh_{12}	Δh_{13}	Δh_{14}	Δh_{AB}	Δh_R
Group 1	1	3.85	1.99	1.17	1.47	1.06	0.88	0.67	0.42	0.34	0.29	0.54	0.41	0.37	0.34	0.12	0.33
Q_{1max} 0.47 m^3/s	0.7	2.70	1.43	0.84	1.06	0.77	0.65	0.51	0.33	0.28	0.24	0.41	0.32	0.29	0.27	0.09	0.24
	0.5	1.94	1.04	0.62	0.78	0.58	0.50	0.40	0.27	0.23	0.20	0.32	0.27	0.24	0.22	0.07	0.18
Group 2	1	3.04	4.81	2.71	3.46	2.44	1.82	1.32	0.72	0.54	0.43	0.93	0.67	0.59	0.54	0.16	0.76
Q_{2max} 0.66 m^3/s	0.7	2.15	3.37	1.91	2.44	1.74	1.31	0.96	0.54	0.42	0.34	0.68	0.51	0.45	0.41	0.12	0.54
	0.5	1.56	2.43	1.39	1.77	1.27	0.97	0.73	0.42	0.33	0.28	0.52	0.39	0.35	0.32	0.10	0.39
Group 3	1	2.24	3.39	5.19	3.23	4.12	1.91	1.48	0.77	0.57	0.45	0.91	0.68	0.59	0.54	0.15	1.62
Q_{3max} 0.89 m^3/s	0.7	1.61	2.42	3.62	2.31	2.92	1.38	1.09	0.58	0.44	0.36	0.68	0.51	0.45	0.41	0.12	1.13
	0.5	1.18	1.76	2.60	1.68	2.12	1.02	0.82	0.45	0.35	0.29	0.52	0.40	0.35	0.33	0.09	0.81
Group 4	1	1.26	1.83	1.38	2.68	1.59	1.70	1.18	0.61	0.45	0.36	0.77	0.55	0.48	0.43	0.12	0.39
Q_{4max} 0.38 m^3/s	0.7	0.90	1.30	0.99	1.88	1.13	1.22	0.86	0.46	0.35	0.29	0.57	0.42	0.36	0.33	0.09	0.32
	0.5	0.66	0.95	0.71	1.36	0.83	0.89	0.64	0.36	0.28	0.24	0.43	0.33	0,29	0.26	0.07	0.24
Group 5	1	1.53	2.29	2.71	2.64	5.42	2.03	1.74	0.84	0.60	0.47	0.91	0.69	0.60	0.55	0.14	1.33
Q_{5max} 0.64 m^3/s	0.7	1.10	1.62	1.90	1.86	3.76	1.45	1.25	0.62	0.46	0.37	0.67	0.52	0.45	0.41	0.11	0.93
	0.5	0.80	1.18	1.38	1.36	2.69	1.06	0.93	0.48	0.36	0.29	0.50	0.40	0.35	0.32	0.09	0.66
Group 6	1	0.49	0.64	0.52	0.93	0.72	1.82	1.25	0.63	0.44	0.36	0.84	0.58	0.48	0.43	0.10	0.19
Q_{6max} 0.20 m^3/s	0.7	0.36	0.47	0.39	0.68	0.52	1.29	0.90	0.47	0.34	0.28	0.62	0.44	0.37	0.33	0.08	0.14
	0.5	0.28	0.36	0.29	0.50	0.40	0.94	0.67	0.37	0.28	0.23	0.47	0.34	0.29	0.26	0.07	0.11
Group 7	1	0.41	0.54	0.46	0.73	0.70	1.34	2.39	1.18	0.70	0.51	0.87	0.81	0.67	0.61	0.12	0.19
Q_{7max} 0.23 m^3/s	0.7	0.31	0.40	0.34	0.54	0.52	0.97	1.69	0.86	0.53	0.39	0.64	0.60	0.50	0.46	0.09	0.14
	0.5	0.25	0.32	0.27	0.42	0.40	0.73	1.27	0.66	0.42	0.32	0.49	0.47	0.39	0.36	0.07	0.11
Group 8	1	0.38	0.46	0.39	0.59	0.55	0.99	1.63	3.30	1.60	1.00	1.11	1.49	1.28	1.27	0.17	0.16
Q_{8max} 0.32 m^3/s	0.7	0.29	0.35	0.29	0.43	0.41	0.72	1.17	2.29	1.14	0.73	0.80	1.06	0.92	0.91	0.13	0.12
	0.5	0.23	0.27	0.23	0.34	0.32	0.54	0.87	1.67	0.85	0.56	0.60	0.79	0.69	0.68	0.10	0.09
Group 9	1	0.44	0.52	0.43	0.65	0.61	1.04	1.59	3.07	5.78	2.55	1.27	1.81	1.82	2.14	0.25	0.17
Q_{9max} 0.50 m^3/s	0.7	0.33	0.39	0.32	0.48	0.46	0.76	1.15	2.16	4.00	1.80	0,91	1.29	1.30	1.52	0.19	0.13
	0.5	0.26	0.31	0.25	0.36	0.35	0.58	0.86	1.57	2.87	1.33	0.69	0.96	0.96	1.11	0.14	0.10
Group 10	1	0.68	0.81	0.67	1.02	0.96	1.61	2.39	3.84	6.22	12.02	2.01	2.89	3.01	3.66	0.44	0.27
Q_{10max} 1.12 m^3/s	0.7	0.50	0.59	0.49	0.74	0.70	1.16	1.70	2.70	4.31	8.13	1.43	2.04	2.12	2.57	0.32	0.20

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(*continued*)

		Δh_1	Δh_2	Δh_3	Δh_4	Δh_5	Δh_6	Δh_7	Δh_8	Δh_9	Δh_{10}	Δh_{11}	Δh_{12}	Δh_{13}	Δh_{14}	Δh_{AB}	Δh_R
	0.5	0.38	0.45	0.37	0.55	0.53	0.85	1.25	1.94	3.07	5.70	1.05	1.48	1.54	1.85	0.24	0.15
Group 11	1	0.33	0.39	0.30	0.50	0.38	0.83	0.70	0.58	0.44	0.36	1.49	0.79	0.64	0.53	0.12	0.11
Q_{11max} 0.18 m^3/s	0.7	0.26	0.30	0.23	0.38	0.29	0.61	0.53	0.44	0.34	0.29	1.07	0.59	0.48	0.40	0.10	0.09
	0.5	0.21	0.24	0.19	0.30	0.24	0.48	0.42	0.35	0.28	0.24	0.81	0.46	0.38	0.32	0.08	0.07
Group 12	1	0.26	0.30	0.24	0.36	0.32	0.59	0.73	0.87	0.64	0.51	1.02	1.52	1.10	0.86	0.14	0.09
Q_{12max} 0.18 m^3/s	0.7	0.21	0.23	0.19	0.28	0.25	0.44	0.55	0.64	0.49	0.40	0.74	1.10	0.80	0.63	0.10	0.07
	0.5	0.17	0.19	0.15	0.23	0.20	0.35	0.43	0.49	0.38	0.32	0.56	0.81	0.60	0.48	0.08	0.06
Group 13	1	0.15	0.16	0.13	0.18	0.16	0.25	0.31	0.40	0.35	0.31	0.36	0.54	0.60	0.50	0.08	0.06
Q_{13max} 0.08 m^3/s	0.7	0.13	0.14	0.11	0.16	0.14	0.21	0.25	0.31	0.28	0.25	0.28	0.41	0.45	0.38	0.07	0.05
	0.5	0.11	0.13	0.10	0.14	0.12	0.18	0.22	0.25	0.23	0.21	0.22	0.32	0.35	0.30	0.06	0.04
Group 14	1	0.14	0.16	0.13	0.18	0.16	0.25	0.31	0.43	0.42	0.36	0.33	0.47	0.51	0.59	0.08	0.06
Q_{14max} 0.08 m^3/s	0.7	0.13	0.14	0.11	0.15	0.14	0.20	0.25	0.33	0.32	0.28	0.26	0.36	0.38	0.44	0.07	0.05
	0.5	0.11	0.13	0.10	0.14	0.12	0.18	0.22	0.27	0.27	0.24	0.22	0.29	0.30	0.34	0.06	0.04

Appendix B:. Coefficients of the regression functions

Based on the data of Appendix A, the tables below contain the coefficients (and the R-square values) of the regression functions:
$1 \le i \le 14$, or $i = AB$, or $i = R$.
$\Delta h_i(x_1, \cdots, x_{14}) = c^i + b^i_1 x_1 + b^i_2 x_2 + \cdots + b^i_{14} x_{14} + a^i_1 x_1^2 + a^i_2 x_2^2 + \cdots + a^i_{14} x_{14}^2$. (A2. 1).

	Δh_1	Δh_2	Δh_3	Δh_4	Δh_5	Δh_6	Δh_7	Δh_8
c^i	−0.22	−0.37	−0.59	−0.37	−0.56	−0.22	−0.20	−0.16
b^i_1	9.27	6.54	5.97	5.44	5.67	3.48	2.98	2.18
b^i_2	5.52	8.59	6.46	6.71	6.05	3.84	3.07	1.98
b^i_3	3.40	5.15	7.52	4.97	6.49	3.05	2.57	1.58
b^i_4	5.08	7.63	7.93	9.65	8.33	6.33	4.92	3.18
b^i_5	3.52	5.29	6.79	5.84	10.59	4.33	3.86	2.28
b^i_6	5.48	8.10	10.26	9.34	10.77	11.17	8.68	5.60
b^i_7	4.82	7.07	9.10	7.95	9.91	8.84	12.97	7.59
b^i_8	3.50	5.02	6.55	5.41	6.92	5.49	7.44	11.88
b^i_9	2.26	3.241	4.121	3.41	4.40	3.52	4.49	6.82
b^i_{10}	1.26	1.74	2.12	1.92	2.35	2.12	2.78	3.88
b^i_{11}	5.96	8.45	11.08	9.12	11.23	8.90	8.06	6.58
b^i_{12}	5.50	7.83	10.63	8.36	10.78	7.45	8.14	8.03
b^i_{13}	10.80	16.45	22.74	16.76	22.38	12.79	13.08	12.11
b^i_{14}	10.10	16.445	22.74	16.57	22.38	12.59	13.08	12.61
a^i_1	−1.21	−3.12	−4.55	−3.14	−4.58	−2.37	−2.36	−2.00
a^i_2	−0.95	−1.23	−2.21	−1.43	−2.27	−1.17	−1.17	−0.99
a^i_3	−0.70	−1.00	−1.10	−1.10	−1.33	−0.71	−0.75	−0.60
a^i_4	−3.06	−4.74	−6.70	−4.19	−6.83	−3.29	−3.34	−3.02
a^i_5	−1.19	−1.73	−2.48	−1.73	−1.88	−1.24	−1.27	−1.11
a^i_6	−9.73	−15.26	−22.96	−14.58	−21.69	−6.66	−8.26	−8.72
a^i_7	−8.98	−13.42	−19.20	−13.77	−19.03	−9.66	−9.12	−8.38
a^i_8	−4.96	−7.38	−10.56	−7.34	−10.47	−5.21	−5.28	−3.27
a^i_9	−1.84	−2.87	−4.04	−2.68	−4.01	−2.01	−1.88	−0.95
a^i_{10}	−0.40	−0.60	−0.86	−0.59	−0.86	−0.42	−0.41	−0.27
a^i_{11}	−15.77	−22.93	−33.18	−23.34	−32.57	−16.87	−16.86	−13.74
a^i_{12}	−15.36	−22.27	−32.46	−23.39	−31.85	−16.15	−16.35	−12.87
a^i_{13}	−75.58	−120.51	−167.61	−121.27	−163.47	−85.33	−83.50	−64.17
a^i_{14}	−79.48	−120.51	−167.61	−119.20	−163.47	−83.26	−83.50	−66.00
R^2	1.00	1.00	0.99	1.00	0.99	1.00	1.00	1.00

	Δh_9	Δh_{10}	Δh_{11}	Δh_{12}	Δh_{13}	Δh_{14}	Δh_{AB}	Δh_R
c^i	−0.18	−0.11	−0.06	−0.07	−0.04	−0.05	0.03	0.01
b^i_1	2.19	1.66	1.85	1.69	1.41	1.38	0.16	0.76
b^i_2	1.84	1.42	1.89	1.56	1.33	1.27	0.21	1.14
b^i_3	1.44	1.11	1.47	1.19	1.01	0.99	0.15	1.78
b^i_4	2.96	2.30	2.87	2.42	1.98	1.92	0.20	1.48
b^i_5	2.04	1.53	1.95	1.67	1.39	1.32	0.21	2.02
b^i_6	5.20	3.84	5.33	4.27	3.51	3.33	0.40	0.99
b^i_7	5.96	4.31	5.10	5.02	4.04	3.93	0.32	0.92
b^i_8	7.13	4.77	4.55	5.76	4.96	4.96	0.48	0.58
b^i_9	11.62	5.74	3.08	4.13	4.00	4.60	0.45	0.40
b^i_{10}	5.86	9.93	2.09	2.86	2.90	3.47	0.39	0.28

(*continued on next page*)

(*continued*)

	Δh_9	Δh_{10}	Δh_{11}	Δh_{12}	Δh_{13}	Δh_{14}	Δh_{AB}	Δh_R
b^i_{11}	6.22	4.84	10.11	6.52	5.22	4.70	0.65	0.84
b^i_{12}	7.38	5.76	7.36	10.20	7.52	6.38	0.48	0.67
b^i_{13}	12.71	9.93	8.20	10.85	10.66	9.88	0.92	0.99
b^i_{14}	13.52	10.59	8.40	10.20	9.31	10.68	0.92	0.99
a^i_1	−2.25	−1.68	−1.19	−1.43	−1.12	−1.15	0.08	−0.15
a^i_2	−1.12	−0.90	−0.60	−0.67	−0.57	−0.57	−0.03	−0.03
a^i_3	−0.65	−0.53	−0.42	−0.39	−0.33	−0.36	−0.02	0.04
a^i_4	−3.36	−2.74	−1.78	−2.08	−1.60	−1.68	0.11	−1.23
a^i_5	−1.26	−0.96	−0.66	−0.76	−0.60	−0.59	−0.05	0.08
a^i_6	−10.49	−7.43	−4.79	−5.54	−4.82	−4.84	−0.27	−0.56
a^i_7	−9.42	−7.08	−5.02	−5.69	−4.46	−4.85	0.28	−0.66
a^i_8	−4.76	−3.98	−2.70	−2.75	−2.55	−2.55	−0.09	−0.31
a^i_9	0.17	−1.00	−0.90	−0.87	−0.68	−0.57	−0.02	−0.16
a^i_{10}	−0.12	0.82	−0.21	−0.19	−0.16	−0.14	−0.02	−0.04
a^i_{11}	−15.14	−12.12	−8.41	−9.78	−8.02	−8.12	−0.77	−1.44
a^i_{12}	−15.39	−12.63	−7.53	−7.70	−6.60	−7.30	0.76	−1.19
a^i_{13}	−74.95	−57.73	−36.32	−41.09	−33.69	−37.55	−3.43	−3.99
a^i_{14}	−74.72	−58.53	−43.89	−43.96	−30.82	−33.65	−3.43	−3.99
R^2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

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