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Estimating the volume of forest growing stock using auxiliary information derived from relascope or ocular assessments

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ABSTRACT

The aim of this study is to demonstrate the potential of integrating probabilistic sampling and estimation with the conventional technique referred to as forest inventory by compartments. The objective of this paper is to propose two strategies for the assessment of growing stock volume using two-phase sampling, namely: (i) relascope basal area estimation performed on first-phase sampling points followed by volume estimation performed on a sub-sample of points selected in the second phase; (ii) ocular evaluation of growing stock volume performed on first-phase sampling plots of fixed size followed by volume estimation performed on a sub-sample of plots selected in the second phase. The effectiveness of using the auxiliary information gathered in the first phase is assessed by comparing the double-expansion estimator of total volume which depends solely on the second-phase sample with the two-phase ratio estimator which instead calibrates the double-expansion estimator on the basis of first-phase information. Conservative estimators of sampling variances and confidence intervals are derived for both the estimators. As is usual in forest inventories, first-phase sampling is assumed to be performed on a systematic random grid while three different schemes are considered for drawing the second-phase sample: simple random sampling without replacement, stratified sampling and 3-P sampling. The performance of double-expansion and ratio estimators under the three schemes adopted in the second phase is empirically checked by means of a simulation study performed on a real compartment in a beech forest of Central Italy. Simulation results show that the use of auxiliary information generated in the first phase constitutes a very effective way of increasing the accuracy of volume estimation at the compartment level, with a moderate increase of fieldwork.

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1. Introduction

Information about the state of and changes to forest stands is fundamental for environmental and timber assessment at various levels of forest management planning (Schreuder et al., 1993; Köhl et al., 2006). Data acquisition is usually based on standwise field inventories which provide updated figures for each forest management unit. Such a survey approach is commonly referred to as *forest inventory by compartments* (Koivuniemi and Korhonen, 2006). In European countries *compartment* means a relatively homogeneous forest area usually about 1–30 ha in size.

Growing stock volume is a key attribute of inventory by compartments. All compartments need to be visited in the field, and the typical problem is the disparity between the required amount of fieldwork and available resources. In countries characterized by high labour costs, the assessment of growing

stock volume at compartment level may be so expensive as to jeopardize management planning.

A solution to this problem has traditionally been found in terms of subjective methods exploiting relascope or ocular assessments (Koivuniemi and Korhonen, 2006). For many decades, these methods have widely replaced those based on full-tree callipering over large tracts, which however still takes place (e.g., in some areas of Alpine and Eastern European countries).

Despite their popularity, subjective inventory methods preclude rigorous evaluations about the uncertainty of estimates and, according to various Authors (e.g., Laasasenaho and Paivinen, 1986; Nersten and Næsset, 1992; Ståhl, 1992; Kinnunen et al., 2007), they are also unreliable. Indeed, as observed by Koivuniemi and Korhonen (2006), even a seemingly highly homogeneous compartment may exhibit considerable variation in growing stock volumes. Accordingly, the forest technician's impression of the average amount of growing stock along a walked route may greatly differ from the actual volume, especially when, as usual, technicians do not visit each part of a compartment. In Scandinavian forests, the accuracy of stand-level inventories for basal area may vary from 13

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to 22% and bias can reach 10% (Haara, 2005), while the accuracy of stand volume estimates may vary from 15 to 45% (Haara, 2005) and the bias may exceed 20% (Nersten and Næsset, 1992; Ståhl, 1992).

Over the last decades, various technological advancements have opened up new opportunities in forest inventories. In particular, the field positioning of sampling units has been greatly facilitated due to the advances and practical availability of global positioning systems. This fact has enhanced the practical suitability of performing inventories via probabilistic procedures based on systematic or random positioning of the sampling units.

The aim of this study is to demonstrate the potential of integrating probabilistic sampling and estimation with conventional techniques of forest inventory by compartments. Two strategies for the estimation of growing stock volume by two-phase sampling are proposed and tested in a complete design-based framework (Gregoire, 1998):

- i. relascope basal area estimation performed on first-phase sampling points followed by volume estimation performed on a sub-sample of points selected in the second phase;
- ii. ocular evaluation of growing stock volume performed on first-phase sampling plots of fixed size followed by volume estimation performed on a sub-sample of plots selected in the second phase.

The paper deals with the theoretical aspects and practical implementation of these two strategies. In Section 2, double-expansion and ratio estimators of total volume at compartment level are considered when the first-phase sampling is performed on a systematic random grid; related sampling variance estimators and confidence intervals are also proposed. For the second-phase sampling, three different schemes are considered: simple random sampling without replacement, stratified sampling and sampling with probability proportional to prediction (usually referred to as 3-P sampling). In Section 3, the performance of double-expansion and ratio estimators is empirically checked for each of the second-phase sampling schemes by means of a simulation study carried out on a real population. Finally, in Section 4, we frame the results in our experience in the perspective of practical applications.

2. Methods

2.1. First-phase sampling

Let U be a population of trees on a delineated study area $\mathcal{A}$ (i.e., a forest compartment) of size A . Denote by y_j the volume of the j th tree, where in accordance with FAO (2004) volume refers to the over bark part of living trees from the stump to the top of the bole, including main branches (see FAO, 2004). It is worth noting that actual volumes are prohibitive to measure and they are usually quantified by means of some standard dendrometric equations predicting the volume as a function of stem diameter at breast height and tree height (van Laar and Akça, 2007). Accordingly, henceforth y_j will denote the predicted volume of the j th tree, rather than its actual volume. Now, suppose the total (predicted) volume

$$TPV = \sum_{j \in U} y_j$$

is the quantity to be estimated by means of an inventory involving N points or plots selected over the study area.

The basic reference sampling scheme to select N points on a study area is *random sampling* in which points are randomly and independently thrown over the area. Even if this scheme is commonly suggested and gives rise to straightforward statistical analysis, it is likely to produce unsuitable voids (i.e., undetected parts) in the study area. Therefore, in order to ensure a systematic search over the area, a stratified scheme is usually preferred by

foresters. One such scheme involves covering the area by means of N rectangles, quadrats or other regular polygons of the same size, each of them containing at least a portion of the study area, and then selecting a point in each of these polygons. If the points are randomly and independently selected within each polygon, the scheme is usually referred to as *unaligned systematic sampling* (see e.g., the EPA, 2002, p. 63), while if a point is randomly selected within one polygon and the other points are selected in the same relative positions in the remaining polygons, the scheme is referred to as *aligned systematic sampling* (see e.g., the EPA, 2002, p. 70).

Barabesi (2003) has theoretically proven that the unaligned systematic scheme provides estimators with variances invariably smaller than those achieved under random sampling, while this cannot be proven for the aligned scheme. Indeed, in the presence of some spatial regularity, the variances arising from the aligned scheme may be even greater than those provided by the random scheme. However, apart from certain situations in which spatial regularities are present, the performance of the aligned scheme is very similar (sometimes superior) to that of the unaligned scheme. The aligned scheme is often adopted by forest technicians since it can be executed by a random shift of a grid superimposed onto a map of the study area, taking the nodes as sample points and locating the points in the terrain by a modern-day GPS system.

On the basis of these considerations, suppose that before starting the survey the study area is covered by a region, say $R \supset A$, of size R and constituted by N non-overlapping quadrats $Q_1, Q_2, \dots, Q_N$ of equal size and such that $Q_i \cap A \neq \emptyset$ for all $i = 1, 2, \dots, N$. Usually, the size of the quadrats is chosen by forest technicians on the basis of practical considerations, while the grid is constructed by using arbitrary orientation and an arbitrary starting point. After the aligned systematic sampling has been performed, a collection of N first-phase points, say $P = \{\mathbf{u}_1, \dots, \mathbf{u}_N\}$, is obtained. Subsequently, Bitterlich sampling or plot sampling are presumed to be carried out in each of these points. It is worth noting that, owing to the costs and time involved, in real situations Bitterlich or plot sampling cannot be performed for each first-phase point, but rather for a portion of these points selected in the second phase. Accordingly, the first-phase survey is only hypothetical and its treatment has the sole aim of constructing the theoretical basis for the subsequent analysis of the second phase.

In the case of Bitterlich sampling, for each first-phase point $\mathbf{u}_i$, all the trees subtending an angle less than a prefixed angle θ onto the point are selected (see e.g., Gregoire and Valentine, 2008, chapter 8). Then, if S_i denotes the set of trees sampled at point $\mathbf{u}_i$, the Horvitz-Thompson estimator of TPV at this points turns out to be

$$Y_i = R \sin^2 \frac{\theta}{2} \sum_{j \in S_i} \frac{y_j}{b_j}, \quad i = 1, \dots, N$$

where b_j denotes the basal area of the tree, expressed in the same unit adopted for R . When, as usual, R is expressed in hectares and the b_j s in m^2 , then the constant $10^4 \sin^2(\alpha/2)$ is named Basal Area Factor (BAF) and is most frequently specified as the characteristic of the measurement device instead of the opening angle θ .

Alternatively, in the case of plot sampling, a circular plot of a pre-fixed size a is centred at each point $\mathbf{u}_i$, in such a way that S_i is constituted by all the trees lying within the circle (see e.g., Gregoire and Valentine, 2008, chapter 7). In this case, the Horvitz-Thompson estimator of TPV at $\mathbf{u}_i$ turns out to be

$$Y_i = \frac{R}{a} \sum_{j \in S_i} y_j, \quad i = 1, \dots, N$$

It is worth noting that some edge effects may be present owing to those trees positioned near the internal edge of the study region, which will have inclusion probabilities smaller than $b_j / \{R \sin^2(\theta/2)\}$ or a/R . A long list of correction methods has been

proposed in order to avoid the negative bias induced by these edge effects (see e.g., Gregoire and Valentine, 2008, Section 7.5). Fortunately, in this framework, the aligned scheme performs like the correction method usually referred to as the *buffer method* (e.g., Gregoire and Valentine, 2008, Section 7.5.1), which entails allowing sample points to fall outside the boundary of A , but within some larger region that includes A . Under the aligned scheme the presence of units whose inclusion zone overlaps the boundary of the enlarged region $R \supset A$ should become negligible. Thus, edge effects can be ignored throughout the paper with no detrimental effect on the bias of the estimator (see also Section 3.4).

Owing to the aligned systematic selection of points, $Y_1, \dots, Y_N$ do not constitute independent realizations of the Horvitz-Thompson estimator. Rather, they are realizations of N correlated random variables, having different expectations $E_P(Y_i)$, different variances $V_P(Y_i)$ and covariances $C_P(Y_i, Y_h)$ for any $h > i = 1, \dots, N$. In this framework, the operators $E_P(\cdot)$, $V_P(\cdot)$ and $C_P(\cdot)$ refer to all the possible sets of points P which can be selected in the first phase by means of aligned systematic sampling. While expectations, variances and covariances of the Y s have cumbersome analytical formulations, by using some analogies with Monte Carlo integration, Barabesi (2003) has shown that their arithmetic mean

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^N Y_i \quad (1)$$

constitutes an unbiased estimator of TPV with variance

$$V_P(\bar{Y}) = \frac{1}{N^2} \sum_{i=1}^N V_P(Y_i) + \frac{2}{N^2} \sum_{h>i=1}^N C_P(Y_i, Y_h) \quad (2)$$

As previously pointed out, nothing ensures that (2) is smaller than the corresponding variance obtained under random sampling. Moreover, since only one point is selected within each quadrat, there is no way to achieve unbiased estimators of (2). This is a common problem with stratified spatial sampling when only one point is selected per stratum. Under the unaligned scheme, owing to the independence among the Y s, their variance divided by N , say

$$\frac{S_Y^2}{N} = \frac{1}{N(N-1)} \sum_{i=1}^N (Y_i - \bar{Y})^2 \quad (3)$$

can be proven to be a conservative estimator for the sampling variance of $\bar{Y}$ (e.g., Wolter, 1985, Theorem 2.4.1). On the other hand, there is no way to prove that (3) is conservative under the unaligned scheme. Nevertheless, owing to the comparability of aligned and unaligned schemes in most situations, they should give rise to very similar variances. Thus, (3) should also provide a basically conservative estimator of (2). Finally, note that for the subsequent second-phase estimation of (3), this quantity can be suitably rewritten as

$$\frac{S_Y^2}{N} = \frac{1}{N^2} \sum_{i=1}^N Y_i^2 - \frac{2}{N^2(N-1)} \sum_{h>i=1}^N Y_i Y_h \quad (4)$$

2.2. Second-phase sampling

In order to reduce survey costs and time, only a portion of points out of the N points selected in the first phase are visited in the second phase to quantify the Y s. Accordingly, the first-phase estimator $\bar{Y}$ is actually unknown and has to be estimated from a second-phase sample. In order to achieve a two-phase estimator of TPV it is worth noting that, conditional on the set of selected points P , the Y s constitute a finite population of N values. Hence, (1) may be viewed as the mean of the finite population $Y_1, \dots, Y_N$ and as such it may be estimated by means of a sample Q of n points selected from P and subsequently visited to achieve the Y s.

Consider a fixed-size sampling scheme adopted to select a sample Q of n points from P before starting the field work or, alternatively, a random-size scheme adopted for the on-site selection of points during the field work. In both cases, the scheme determines the first-order inclusion probability π_i and the second-order inclusion probability π_{ih} for $h > i = 1, \dots, N$. Thus, in accordance with Särndal et al. (1992, chapter 9), the double-expansion estimator of TPV turns out to be

$$\hat{Y} = \frac{1}{N} \sum_{i \in Q} \frac{Y_i}{\pi_i} \quad (5)$$

From the properties of the double-expansion estimator (Särndal et al., 1992, Section 9.3), $\hat{Y}$ turns out to be an unbiased estimator of TPV , in the sense that

$$E_{P \wedge Q}(\hat{Y}) = TPV$$

with variance

$$V_{P \wedge Q}(\hat{Y}) = E_P \{ V_Q(\hat{Y}|P) \} + V_P(\bar{Y}) \quad (6)$$

where the operators $E_{P \wedge Q}(\cdot)$ and $V_{P \wedge Q}(\cdot)$ now refer to both first- and second-phase sampling, the operators $E_Q(\cdot|P)$ and $V_Q(\cdot|P)$ refer to all the possible second-phase samples Q conditional to the set of points P selected in the first phase, and

$$V_Q(\hat{Y}|P) = \frac{1}{N^2} \sum_{i=1}^N \frac{1 - \pi_i}{\pi_i} Y_i^2 + \frac{2}{N^2} \sum_{h>i=1}^N \frac{\pi_{ih} - \pi_i \pi_h}{\pi_i \pi_h} Y_i Y_h \quad (7)$$

constitutes the sampling variance of $\hat{Y}$ conditional to P .

As to the estimation of $V_{P \wedge Q}(\hat{Y})$, if the design ensures $\pi_{ih} > 0$ for any $h > i = 1, \dots, N$, then the first term of (6) can be unbiasedly estimated by the quantity (Särndal et al., 1992, expression 2.8.6)

$$\hat{v}^2 = \frac{1}{N^2} \sum_{i \in Q} \frac{1 - \pi_i}{\pi_i^2} Y_i^2 + \frac{2}{N^2} \sum_{h>i \in Q} \frac{\pi_{ih} - \pi_i \pi_h}{\pi_i \pi_h} \frac{Y_i Y_h}{\pi_{ih}} \quad (8)$$

while, in the case of fixed-size designs, the estimator

$$\hat{v}^2 = \frac{1}{N^2} \sum_{h>i \in Q} \frac{\pi_i \pi_h - \pi_{ih}}{\pi_{ih}} \left(\frac{Y_i}{\pi_i} - \frac{Y_h}{\pi_h} \right)^2 \quad (9)$$

referred to as the Sen-Yates-Grundy estimator is more commonly used (Särndal et al., 1992, expression 2.8.11). Moreover, according to (4) an unbiased estimator of the second term in (6) is given by

$$\frac{\hat{S}_Y^2}{N} = \frac{1}{N^2} \sum_{i \in Q} \frac{Y_i^2}{\pi_i} - \frac{2}{N^2(N-1)} \sum_{h>i \in Q} \frac{Y_i Y_h}{\pi_{ih}} \quad (10)$$

Then, it can be proven that $\hat{V}^2 = \hat{v}^2 + \hat{S}_Y^2/N$ is a conservative estimator of (6), i.e., $E_{P \wedge Q}(\hat{V}^2) > V_{P \wedge Q}(\hat{Y})$, providing that S_Y^2/N is a conservative estimator of $V_P(\bar{Y})$.

2.3. Second-phase sampling with auxiliary variables

When surveys are performed in relatively large areas, all the N first-phase points are usually visited for, at least, a qualitative and informal monitoring of forest attributes. On the other hand, the estimation of TPV , requiring fairly cumbersome measurements, is performed only for the n points of the second-phase sample Q .

If Bitterlich sampling is adopted, the Horvitz-Thompson estimates of the total for basal areas

$$X_i = R \sin^2 \frac{\theta}{2} \{ (S_i) \} \quad (11)$$

can be readily performed on each first-phase point, since it entails no measurement except for the enumeration of those trees subtending the angle θ .

As is known in dendrometric studies (e.g., van Laar and Akça, 2007), a strong linear relationship passing through the origin has been proven between the volumes (y_i) and basal areas (b_i) at tree level. Accordingly, a similar relationship is likely to occur between the X_i s and the Y_i s, in such a way that the X_i s can be adopted as auxiliary information to use in the second phase. Quoting again from dendrometric literature, auxiliary information can be taken into account by means of an assisting model of type

$$Y_i = \beta X_i + \varepsilon_i, \quad i = 1, \dots, N \quad (12)$$

where the ε_i s are assumed to have zero expectations and variances proportional to the X_i s. Model (12) is usually referred to as the *common ratio model* (see e.g., Särndal et al., 1992, section 7.3).

In a similar way, when plot sampling is adopted instead of Bitterlich sampling, an ocular evaluation of the tree volumes within the plots, say X_i , can be easily performed on each first-phase point and the X_i s can serve again as second-phase auxiliary information. Since X_i will be 0 when $Y_i = 0$ (i.e., no trees in the plot) and since evaluation errors should increase in size as the actual volumes increase, a common ratio model could serve to link ocular evaluations and actual volumes (Corona, 2006). Accordingly, it may be reasonable that

$$X_i = \beta^* Y_i + \varepsilon_i^*, \quad i = 1, \dots, N \quad (13)$$

where the ε_i^* s have zero expectations and variances proportional to the Y_i s. Obviously, relation (13) must be reversed to obtain an assisting model similar to (12) with $\beta = 1/\beta^*$ and $\varepsilon_i = -\varepsilon_i^*/\beta^*$. Note however that the resulting model differs from the genuine ratio model since the ε_i s have 0 expectations but the variances are proportional to the Y_i s. However, since X_i increase with Y_i , the variances of the ε_i s may also be considered proportional to the X_i s.

From Särndal et al. (1992, example 6.4.2), the common ratio model (12) leads to the ratio estimator

$$\hat{Y}_{\text{ratio}} = \hat{\beta} \bar{X} \quad (14)$$

where $\bar{X}$ denotes the population mean of the X_i s, $\hat{\beta} = \hat{Y}/\bar{X}$, while

$$\hat{X} = \frac{1}{N} \sum_{i \in Q} \frac{X_i}{\pi_i}$$

denotes the double-expansion estimator for $\bar{X}$. Quoting again from Särndal et al. (1992, example 6.6.1), the ratio estimator is approximately unbiased, in the sense that $E_Q(\hat{Y}_{\text{ratio}}|P) \approx \bar{Y}$, with approximate variance

$$V_Q(\hat{Y}_{\text{ratio}}|P) \approx \frac{1}{N^2} \sum_{i=1}^N \frac{1-\pi_i}{\pi_i} e_i^2 + \frac{2}{N^2} \sum_{h>i=1}^N \frac{\pi_{ih} - \pi_i \pi_h}{\pi_i \pi_h} e_i e_h \quad (15)$$

where $e_i = Y_i - \hat{\beta} X_i$.

Accordingly, $\hat{Y}_{\text{ratio}}$ turns out to be an approximately unbiased estimator of TPV , in the sense that

$$E_{P \wedge Q}(\hat{Y}_{\text{ratio}}) \approx TPV$$

with an approximate overall variance

$$V_{P \wedge Q}(\hat{Y}_{\text{ratio}}) \approx E_P\{V_Q(\hat{Y}_{\text{ratio}}|P)\} + V_P(\bar{Y}) \quad (16)$$

Finally, as to the estimation of $V_{P \wedge Q}(\hat{Y}_{\text{ratio}})$, if the design ensures $\pi_{ih} > 0$ for any $h > i = 1, \dots, N$, then, as is customary in ratio estimation (Särndal et al., 1992, example 6.6.1), the first term of (16) can be estimated by means of

$$\hat{v}_{\text{ratio}}^2 = \left(\frac{\bar{X}}{\hat{X}}\right)^2 \frac{1}{N^2} \sum_{i \in Q} \frac{1-\pi_i}{\pi_i^2} \hat{e}_i^2 + 2 \left(\frac{\bar{X}}{\hat{X}}\right)^2 \frac{1}{N^2} \sum_{h>i \in Q} \frac{\pi_{ih} - \pi_i \pi_h}{\pi_i \pi_h} \frac{\hat{e}_i \hat{e}_h}{\pi_{ih}} \quad (17)$$

or, in the case of fixed-size designs, by means of

$$\hat{v}_{\text{ratio}}^2 = \left(\frac{\bar{X}}{\hat{X}}\right)^2 \frac{1}{N^2} \sum_{h>i \in Q} \frac{\pi_i \pi_h - \pi_{ih}}{\pi_{ih}} \left(\frac{\hat{e}_i}{\pi_i} - \frac{\hat{e}_h}{\pi_h}\right)^2 \quad (18)$$

where $\hat{e}_i = Y_i - \hat{\beta} X_i$.

Since the second-phase estimator of the second term in (16) is identical to (10), the quantity $\hat{V}_{\text{ratio}}^2 = \hat{v}_{\text{ratio}}^2 + \hat{S}_Y^2/N$ is again conservative when $\hat{S}_Y^2/N$ is a conservative estimator of $V_P(\bar{Y})$.

2.4. Second-phase schemes

Three schemes for the selection of second-phase samples are considered: simple random sampling without replacement (SRSWOR), stratified sampling (STRS) and 3-P sampling (3PS). The first two fixed-size schemes involve the selection of Q before starting the field work while 3PS is a random-size scheme and involves on-site selections during the field work.

In the case of SRSWOR, n points are randomly selected without replacement from P ; the first- and second-order inclusion probabilities under SRSWOR are

$$\pi_i = \frac{n}{N}$$

and

$$\pi_{ih} = \frac{n(n-1)}{N(N-1)}$$

(see e.g., Särndal et al., 1992, Example 2.4.1). Alternatively, in order to achieve a more even distribution of second-phase points over the study area, the N first-phase points may be partitioned into L spatial strata of equal size. Subsequently, n/L points are selected within each stratum by means of SRSWOR. In this case the first-order inclusion probabilities are the same as with SRSWOR, while the second-order probabilities are

$$\pi_{ih} = \frac{n(n-L)}{N(N-L)}$$

when points i and h are in the same stratum and

$$\pi_{ih} = \frac{n^2}{N^2}$$

when points i and h are in different strata.

To perform 3PS, the largest possible X_i , say X_{MAX} is presumed and a constant $C > X_{\text{MAX}}$ is chosen. Then, during the field work, following the recording of each auxiliary value X_i , the point is selected with probability

$$\pi_i = \frac{X_i}{C}$$

In 3PS sample size n is a random variable with expectation

$$E_{P \wedge Q}(n) = E_P\{E_Q(n|P)\} = E_P\left\{\frac{N\bar{X}}{C}\right\}$$

Moreover, since the selection of points is performed independently, the second-order inclusion probabilities are

$$\pi_{ih} = \pi_i \pi_h = \frac{X_i X_h}{C^2}$$

For more details on 3PS, see e.g., De Vries (1986, chapter 15) and Gregoire and Valentine (2008, Section 11.7).

First- and second-order inclusion probabilities of these schemes are inserted in the expressions (5), (8), (9) and (10) to give the double-expansion estimator $\hat{Y}$ and its variance estimators or into expressions (14), (17) and (18) to obtain the ratio estimator $\hat{Y}_{\text{ratio}}$ and its variance estimators.

3. A Monte Carlo experiment

3.1. Materials

The performance of double-expansion and ratio estimators under SRSWOR, STRS and 3PS was checked by means of simulating sampling in a real population. The study area $\mathcal{A}$ was a forest compartment of size $A = 11.934$ ha located within the beech (*Fagus sylvatica* L.) forest of Monte Cimino (Central Italy). For a detailed description of the study area, see Lamonaca et al. (2008) (Fig. 1).

A complete field enumeration gave tree volumes (y_j) and basal areas (b_j) for the whole population of 2734 trees and $TPV = 9049.313 \text{ m}^3$. Fig. 2 plots tree basal areas against their volumes. An investigation performed by a MSc Student on the beech forest including the study area gave actual values (Y_i) and ocular evaluations (X_i) of stand volumes in 37 plots centered on a systematic grid of points; Fig. 3 plots ocular evaluations against

Fig. 1. Graphical representation of the study area and covering region. Marked points within the study area represent tree locations.

Fig. 2. Scatterplot of basal areas (b_j) against volumes (y_j) for the 2734 trees within the study area.

Fig. 3. Scatterplot of ocular evaluations (X_i) against true volumes (Y_i) in 37 plots centered on a systematic grid of points superimposed onto the beech forest including the study area.

true volumes. Both the scatterplots support the common ratio model in (12).

3.2. Experiment setting

To perform unaligned systematic sampling, the study area was covered by a quadrat region $\mathcal{R}$ of size $\mathcal{R} = 16$ ha (side 400 m). To simulate an intensive inventory, $\mathcal{R}$ was partitioned into $N = 64$ quadrats of size 0.25 ha (side 50 m) (see Fig. 1) and 10,000 first-phase samples were randomly generated by means of unaligned systematic sampling. Accordingly, each simulated first-phase sample P had $N = 64$ points in which one point was randomly selected in a quadrat (e.g., the first quadrat) with the other points systematically repeated in the $N - 1$ remaining quadrats.

For each point $\mathbf{u}_i \in P$ the value of the auxiliary variable was determined. In Bitterlich sampling, X_i is the basal area estimate obtained from the trees subtending an angle less than $\theta = 0.02828$ radians ($=1.6206^\circ$) onto $\mathbf{u}_i$. Such an angle corresponds to a BAF of 2, which is commonly used in forest inventories. For this BAF, the basal area estimate in (11) expressed in m^2 reduces to $X_i = 2 \times R \times \#(S_i)$. In plot sampling, the auxiliary variable X_i is the ocular evaluation of total volume within the plot of radius 15 m (size $a = 706.858 \text{ m}^2$) centred at $\mathbf{u}_i$. Evaluations were generated on the basis of the following assumptions: (a) the evaluations were fixed for a given $\mathbf{u} \in \mathcal{R}$; (b) also, similar evaluations were expected for points $\mathbf{u}_1, \mathbf{u}_2$ in close proximity; (c) evaluation errors increased in size as the actual volume increased. Accordingly, the evaluation at a point $\mathbf{u} \in \mathcal{R}$ was assumed to be a deterministic continuous function of $\mathbf{u}$ of type

$$X(\mathbf{u}) = kY(\mathbf{u}) + \beta Y(\mathbf{u})\varphi(\mathbf{u}) \quad (19)$$

where k and β represented positive parameters, $Y(\mathbf{u})$ denoted the actual predicted volume within the plot of radius 15 m centred at $\mathbf{u}$,

$$\varphi(\mathbf{u}) = \frac{a(v_1) + a(v_2) + a(v_1v_2)}{6}$$

represented a fluctuation in the range $-1, 1$, v_1 and v_2 constitute the two coordinates of $\mathbf{u}$ rescaled as to vary between 0 and 1 and

$$a(v) = \sin\left(-\frac{\pi}{2} + v\pi\right) + \cos(v\pi)$$

Practically speaking, the ocular evaluation of volume was assumed to be a linear function of the actual volume in the plot plus a positive or negative perturbation increasing in size with the actual volume. In order to allow situations where a forest technician tends to over-evaluate or under-evaluate the actual volume (Corona, 2006), k was set equal to 1.2 and 0.8 while β was set to 0.8 throughout.

After X_{1s} were determined for the $N = 64$ first-phase points, a second-phase sample Q of size $n = 8, 16$ was selected from P by means of SRSWOR, STRS with $L = 4$ spatial strata determined by the four quadrats of size 4 ha (side 200 m) partitioning $\mathcal{R}$ and 3PS. The value of C in 3PS was chosen to give the same average sample size as in SRSWOR and STRS.

3.3. Outputs

For each simulated second-phase sample Q , the following statistics were computed: double-expansion estimate $\hat{Y}$, variance estimate $\hat{V}^2$, relative standard error estimate $R\hat{S}E = \hat{V}/\hat{Y}$, 0.95 confidence interval $\hat{Y} \pm 1.96\hat{V}$; and its width $W = 1.96\hat{V}$, ratio estimate $\hat{Y}_{ratio}$, variance estimate $\hat{V}_{ratio}^2$, relative standard error estimate $R\hat{S}E_{ratio} = \hat{V}_{ratio}/\hat{Y}_{ratio}$, 0.95 confidence interval $\hat{Y}_{ratio} \pm 1.96\hat{V}_{ratio}$; and its width $W_{ratio} = 1.96\hat{V}_{ratio}$.

Monte Carlo distributions of the above statistics were used to derive the following expectations: $E_{P \wedge Q}(\hat{Y})$, $MSE_{P \wedge Q}(\hat{Y}) = E_{P \wedge Q}\{(\hat{Y} - TPV)^2\}$, $E_{P \wedge Q}(R\hat{S}E)$, $E_{P \wedge Q}(W)$, $E_{P \wedge Q}(\hat{Y}_{ratio})$, $MSE_{P \wedge Q}(\hat{Y}_{ratio}) = E_{P \wedge Q}\{(\hat{Y}_{ratio} - TPV)^2\}$, $E_{P \wedge Q}(R\hat{S}E_{ratio})$, $E_{P \wedge Q}(W_{ratio})$.

In turn, the above expectations allowed for the computation of: relative bias (RB), $\{E_{P \wedge Q}(\hat{Y}) - TPV\}/TPV$ and $\{E_{P \wedge Q}(\hat{Y}_{ratio}) - TPV\}/TPV$, relative root mean squared error (RRMSE), $\sqrt{MSE_{P \wedge Q}(\hat{Y})}/TPV$ and $\sqrt{MSE_{P \wedge Q}(\hat{Y}_{ratio})}/TPV$, expected relative standard error estimate (ERSEE), $E_{P \wedge Q}(R\hat{S}E)$ and $E_{P \wedge Q}(R\hat{S}E_{ratio})$ and expected confidence interval relative width (ECIRW), $E_{P \wedge Q}(W)/TPV$ and $E_{P \wedge Q}(W_{ratio})/TPV$. Finally the actual coverage (AC) of the confidence intervals was determined as the percentage of times the intervals contained the true value of TPV .

3.4. Results

Tables 1–3 report the percent values of RB, RRMSE, ERSEE, AC and ECIRW for both Bitterlich and plot sampling (involving ocular over- and under-evaluations) performed on $N = 64$ first-phase points or plots, with $n = 8, 16$ points selected in the second phase by means of SRSWOR. The results obtained using STRS and 3PS are not reported here for brevity, since they are completely equivalent to those achieved with SRSWOR.

There is negligible bias in results with double-expansion and ratio estimators. Absolute values of RB are always less than 0.3% and, in most cases, less than 0.1%. In all settings the ratio estimator shows important reduction of RRMSEs when compared to the

Table 1

Performance of double-expansion and ratio estimators in the case of Bitterlich sampling performed on $N = 64$ first-phase sampling points (BAF = 2). Second-phase scheme: SRSWOR.

Sample size	8		16	
	Double-expansion estimator	Ratio estimator	Double-expansion estimator	Ratio estimator
RB (%)	−0.23%	−0.05%	−0.07%	−0.11%
RRMSE (%)	18.05%	3.90%	11.93%	2.98%
ERSEE (%)	19.27%	7.53%	13.59%	7.13%
AC (%)	91.78%	99.81%	95.72%	100.00%
ECIRW (%)	73.44%	28.90%	52.73%	27.68%

Table 2

Performance of double-expansion and ratio estimators in the case of plot sampling with ocular evaluations performed on $N = 64$ first-phase plots of radius 15 m. Ocular evaluations are determined by means of Eq. (19) with $k = 1.2$ (over-evaluations). Second-phase scheme: SRSWOR.

Sample size	8		16	
	Double-expansion estimator	Ratio estimator	Double-expansion estimator	Ratio estimator
RB (%)	−0.18%	0.23%	−0.00%	−0.01%
RRMSE (%)	23.69%	8.30%	16.30%	7.44%
ERSEE (%)	24.24%	9.53%	17.09%	9.04%
AC (%)	91.00%	95.61%	93.78%	97.77%
ECIRW (%)	92.42%	36.72%	66.22%	35.07%

Table 3

Performance of double-expansion and ratio estimators in the case of plot sampling with ocular evaluations performed on $N = 64$ first-phase plots of radius 15 m. Ocular evaluations are determined by means of Eq. (19) with $k = 0.8$ (under-evaluations). Second-phase scheme: SRSWOR.

Sample size	8		16	
	Double-expansion estimator	Ratio estimator	Double-expansion estimator	Ratio estimator
RB (%)	−0.18%	0.53%	−0.00%	0.12%
RRMSE (%)	23.69%	9.86%	16.30%	8.20%
ERSEE (%)	24.24%	10.57%	17.09%	9.60%
AC (%)	91.00%	94.09%	93.78%	97.23%
ECIRW (%)	92.42%	40.88%	66.22%	37.30%

double-expansion estimator. In contrast to the double-expansion estimator, the ratio estimator always gave conservative estimates of RRMSE and confidence intervals with actual coverage which are above the nominal level of 95%. Moreover, the confidence intervals constructed around ratio estimates are much shorter than intervals obtained via double-expansion estimates.

4. Discussion and conclusion

Inventory by compartments at 10–15 year intervals is still very common method for conducting surveys in support of forest management planning in Europe. In some countries, large forest companies and forest enterprises appear to prefer continuous inventory (Koivuniemi and Korhonen, 2006). The basic ideas behind inventory by compartments originated in Central Europe long before probabilistic sampling theory was developed for this type of estimation problem. More recently many authors (e.g., Laasasenaho and Paivinen, 1986; Ståhl, 1992; Haara, 2005; Kinnunen et al., 2007) have acknowledged that errors in compartment-level estimates can be reduced if supplemented with probability sampling.

The results from the Monte Carlo experiment highlight the importance of using probabilistic sampling in the estimation of growing stock volume at the compartment level. Indeed, bias has proved to be negligible for both double-expansion and ratio estimators as well as under Bitterlich and plot sampling and for any second-phase scheme. This finding is of particular interest with respect to conventional inventories by compartments, in which discrepancies over 10% are frequently observed between single guess/ocular assessments and census results (e.g., Nersten and Næsset, 1992; Ståhl, 1992; Kinnunen et al., 2007). Our results have demonstrated how easy and fast auxiliary data can improve the efficiency of probabilistic volume sampling.

Basal area estimates (Bitterlich sampling) provides more accurate estimation than ocular evaluations (plot sampling). Notwithstanding, ocular evaluation remains an option owing to

its great practical feasibility (e.g., Rondeaux et al., 1998; Bousson et al., 2002; Kangas et al., 2004; Lejeune et al., 2005). Ocular evaluations may also work reasonable well with less than fully experienced technicians as long as they are consistent in their calls. Indeed, ocular evaluations are only adopted to calibrate the second-phase estimate, so that under- or over-evaluations do not greatly affect the bias of the final ratio estimate. Instead, evaluation errors do more harm to the accuracy of the ratio estimator than the use of objective information such as basal area estimates.

A critical aspect of the proposed strategies is the need for rigorous geo-location of the sampling units. Fortunately, it has been greatly facilitated over the last decades. A rigorous location of points is especially critical for the Bitterlich method; even nearby point locations may produce substantially different estimates. Ocular plot sampling is more robust against location errors; a certain tolerance may be admitted in positioning the sampling units.

In relatively large compartments containing, for examples, a mix of different species, silvicultural treatments or age structures, a stratification may be necessary to improve the efficiency of the sampling design. In that case reference should be made to the proper inclusion probabilities of second-phase units (see Section 2.4).

With efficiently updated inventory by compartment, strategic and tactical planning and decisions can be based on the timely knowledge of local forest resources. Operative planning and subsequent activities may become more cost-efficient. On the basis of Italian experience (Corona and Ferrara, 1989; Corona, 2006), the total time required by a two-technician team to perform the two-phase inventory described in the simulation is about a fourth of the time needed to calliper all the trees and measure a sample of heights or about twice the time necessary to perform first-phase ocular or relascope assessment. Two-phase inventories of local forest attributes provide a compromise between very intensive field assessments in which all the first-phase points are considered and reliance on auxiliary data correlated with the variable of interest but not sufficient for the specific inventory task. Auxiliary data such as the basal area or ocular evaluations can indeed be adopted to assist and improve the design-based estimation of forest volumes but they are not sufficient to provide a direct estimation, unless models relating the auxiliary data to volumes are introduced. In these cases the inference is of a model-based nature and the resulting estimators are strictly dependent on model validity.

From a practical point of view, a two-phase sampling is attractive when (i) the unit cost of acquiring first-phase data is significantly lower than that of second-phase data; (ii) the auxiliary information collected in the first phase is well correlated with the estimates performed at second-phase points. Both these requirements are met in the present study. As such, the strategies proposed in the paper seem to merit further investigation to explore their effectiveness under various compositional, structural and silvicultural conditions.

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